The Zeeman polarised spectral synthesis programme

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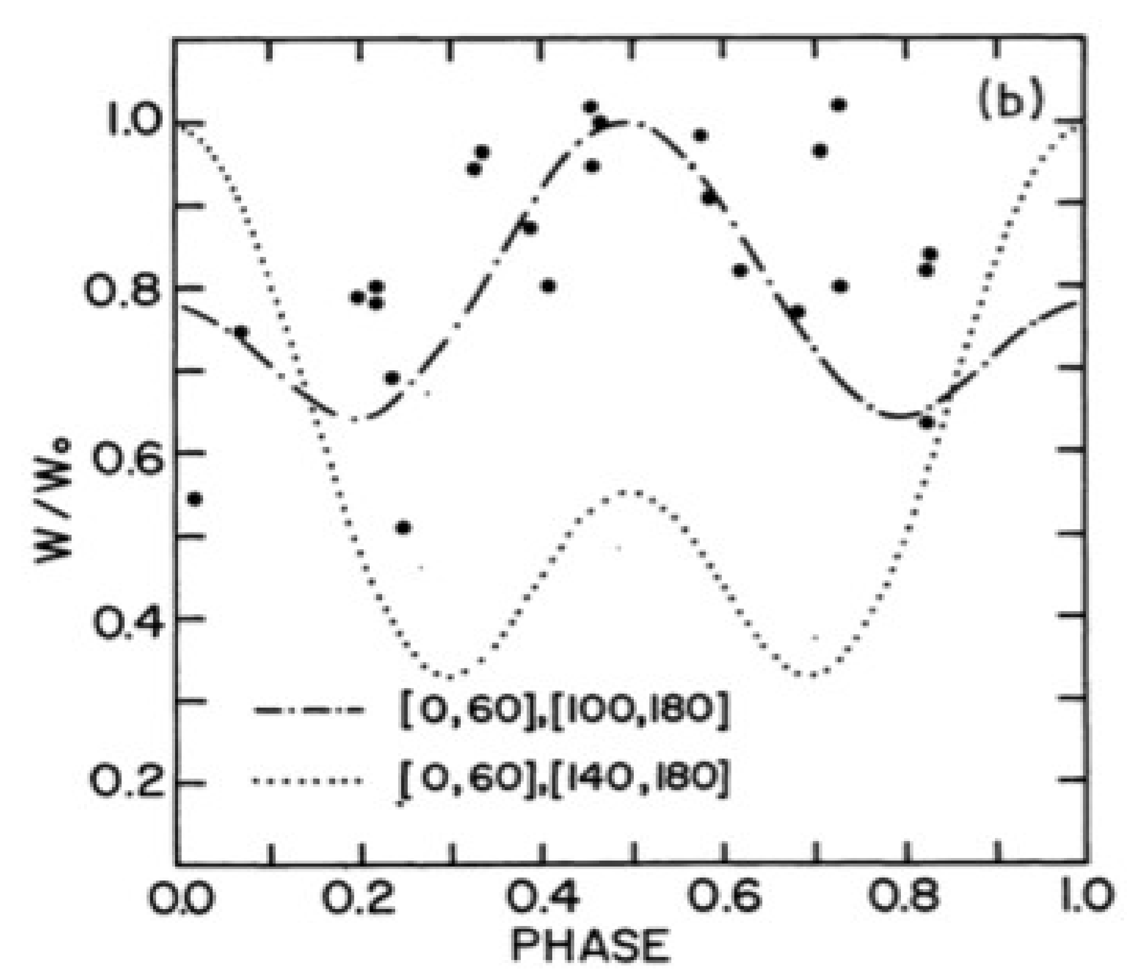
Introduction

- Zeeman arose from fact that theory of diffusion in Ap atmospheres had hardly any suitable data to confront
- Zeeman is by now a rather old code. It was the first code for stellar spectrum synthesis with polarised radiative transfer (mid 1980's), but was based on previous solar codes, especially Wittmann's
- Zeeman's main objective was to explore question of what one could learn from high-resolution intensity (*I*) spectra together with Balmer-line magnetic measurements
- Intention was to see if simple mapping of field and abundances simultaneously would reproduce observations and provide solid data about (a) field structure and (b) abundances (on reliable scale) and their gross variations over stellar surface
- We now know that this problem is much harder than I imagined in 1984!

Zeeman and CFHT

- You could say that Zeeman was created to make it possible to move on from data like these, which clearly don't define a model very precisely.
- The object was to exploit the kind of high precision *I* data that were being produced by the Gordon Walker's Reticon on the f/8 spectrograph at CFHT in early 1980's



Design of Zeeman

- Zeeman was designed around a number of criteria
  - It had to run in a resonable time on machines of the day (IBM 360, Cybers, ETAs), so FORTRAN77 was a natural choice
  - I intended it to incorporate as much realistic physics as possible for polarised line formation, so that derived abundances would be as reliable as possible
  - It was *not* intended to do detailed mapping in the sense of Zeeman-Doppler imaging; this would initially have been completely incompatible with the two criteria above, and I have never moved the code in the direction of such mapping
  - It was completely written from scratch so that it could easily be modified to do experiments as needed. To a considerable extent the code is divided into natural subroutines that may be replaced individually
  - It is heavily documented; more than 20% of ~4000 lines are comments

Structure of (current) Zeeman

- Start of code describes overall structure and revisions
- Programme is designed as a fitting routine to make comparison with one or several spectra in up to 3 spectral windows, and (if desired) to optimise abundance of one element at a time
- Programme assumes a simple multipole field, with inclination of rotation axis, obliquity and strength of field specified
- Model atmosphere interpolated from precomputed ATLAS grid
- Programme is given observed spectra and corresponding (VALD) list of lines to use for fitting, together with other needed data, and an initial assumed abundance distribution
- It then computes polarised spectra corresponding to observations provided: (I, Q, U, V)
- Automatically fits v_{rad}, v sin i, and (if desired) abundances (by iteration) on up to 6 rings concentric about field axis for one element

Data requirements

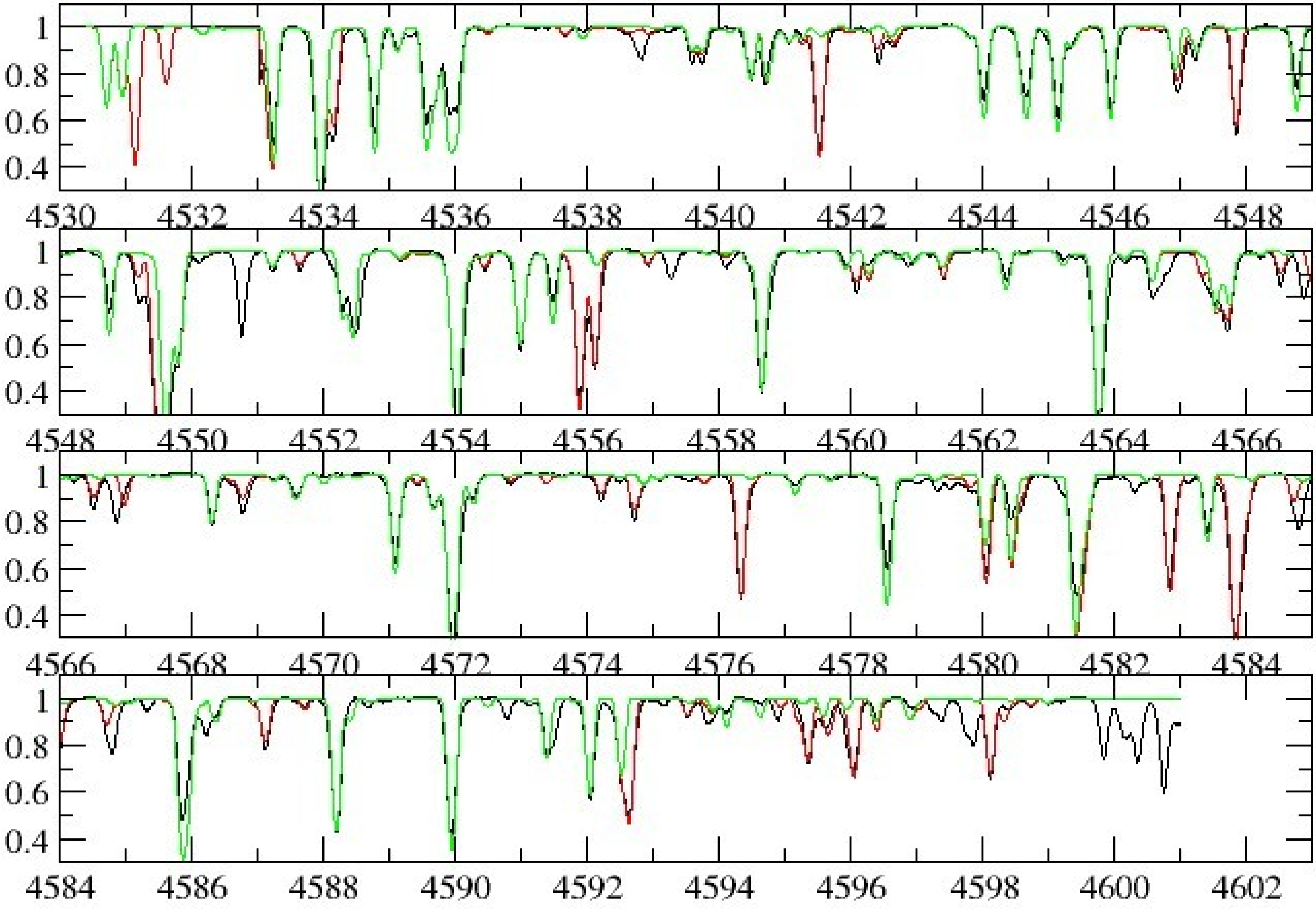
- Model atmosphere: in principle, should be iterated along with abundances determined from lines. Now possible using LLModels by Khan & Shulyak, but this is not implimented in Zeeman. Current version uses pre-computed ATLAS solar abundance model grid
- Line data: need best available gf values and Lande factors. VALD provides an extremely valuable resource
    - But the gf values sometimes need to be tuned using suitable spectra (I prefer normal A stars: well mixed atmosphere, but because convection is inefficient, T(z) well determined)
    - Many Lande factors in VALD are from Atomic Energy Levels (Moore, NBS): good experimental values; others from Kurucz
    - When missing, can use LS or other coupling rules
- Continuous opacities include fairly accurate H, approximate He, and some metals in very approximate form (mostly important around 1200 A or below)
- Partition functions are an important source of uncertainty for non-dominant ions!

Optimisation

- The forward calculation of a spectrum is straight-forward.
- Comparison with observed spectra:
  - Initially, the spectrum of the desired element is computed for “expected” abundance, and very low abundance. From difference spectrum, “clean” lines are identified and marked
  - At most iterations of fit, only marked lines are re-computed, so chi-square of fit is only calculated for these lines
- Optimisation involves
  - Computing a set of forward spectra with various abundance “vectors” (vector describes the abundance values on each of several rings)
  - Comparing with observed spectra
  - Choosing a new abundance vector for computation by downhill simplex method
  - Recomputing forward spectrum, repeating comparison
  - Repeating cycle until convergence (or at least local minimum)

HD 61421 = Procyon: F5 IV-V

Identification of clean lines of Fe (in red)

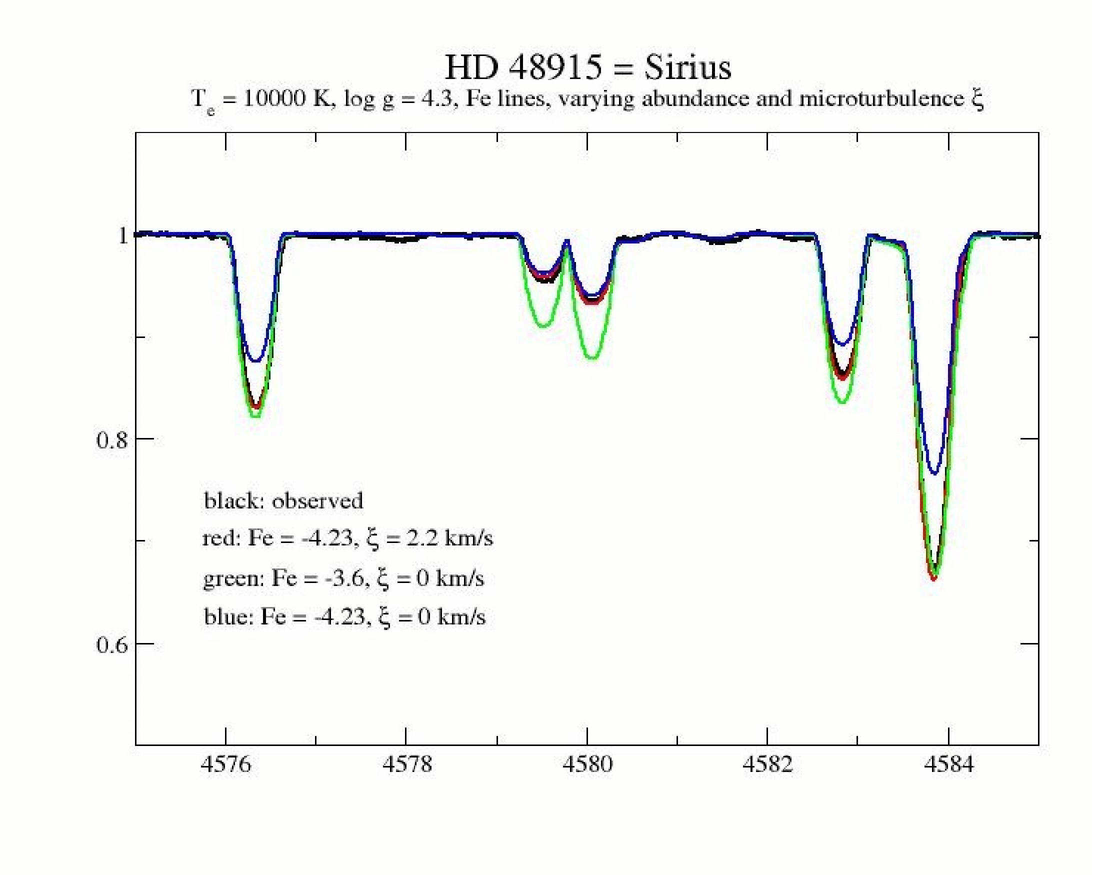


Fitting non-magnetic spectra

- Some of magnetic field physics may be turned off, allowing Zeeman to model non-magnetic stars
- This reduces cpu time by factor of order 30 or more
- Spectral lines are still treated as made up of many Zeeman components, so code is slow compared to normal non-magnetic synthesis code
- Automated iteration system is same for both magnetic and non-magnetic cases
- In non-magnetic case, code may be used to optimise v_{rad}, v sin i, microturbulence parameter ξ, and abundance of one element at a time
- Code has been quite useful at identifying signatures of atmospheric velocity field in spectra of sharp-line A and B stars

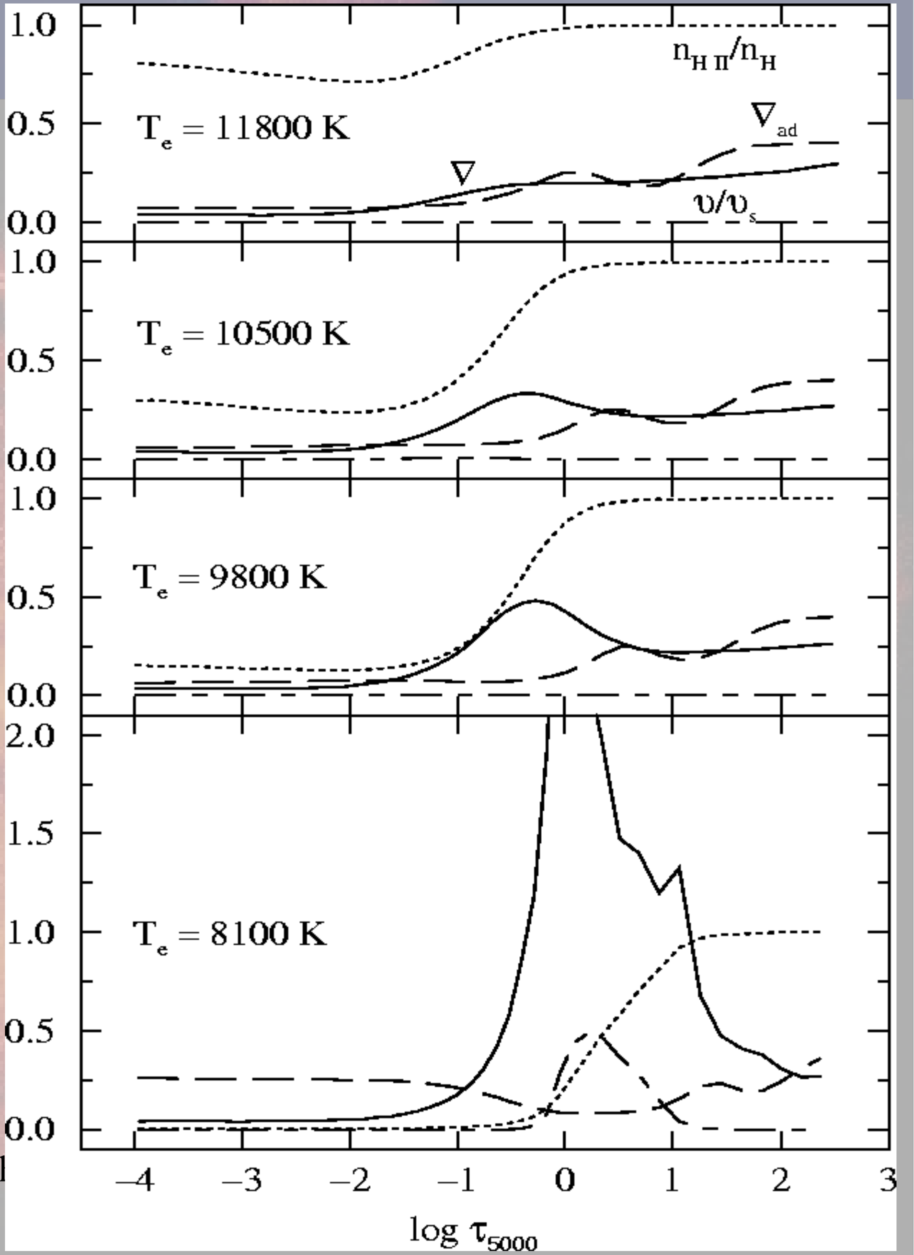
What is this “microturbulence”?

- If we try to fit spectrum using “expected” thermal broadening of local line profiles, we fail to fit both weak and strong lines with same abundance
- Problem seems to be that local line widening is more than expected, which does not affect weak lines but spreads absorption of strong lines over wider window: desaturates them
- Makes them stronger for given abundance



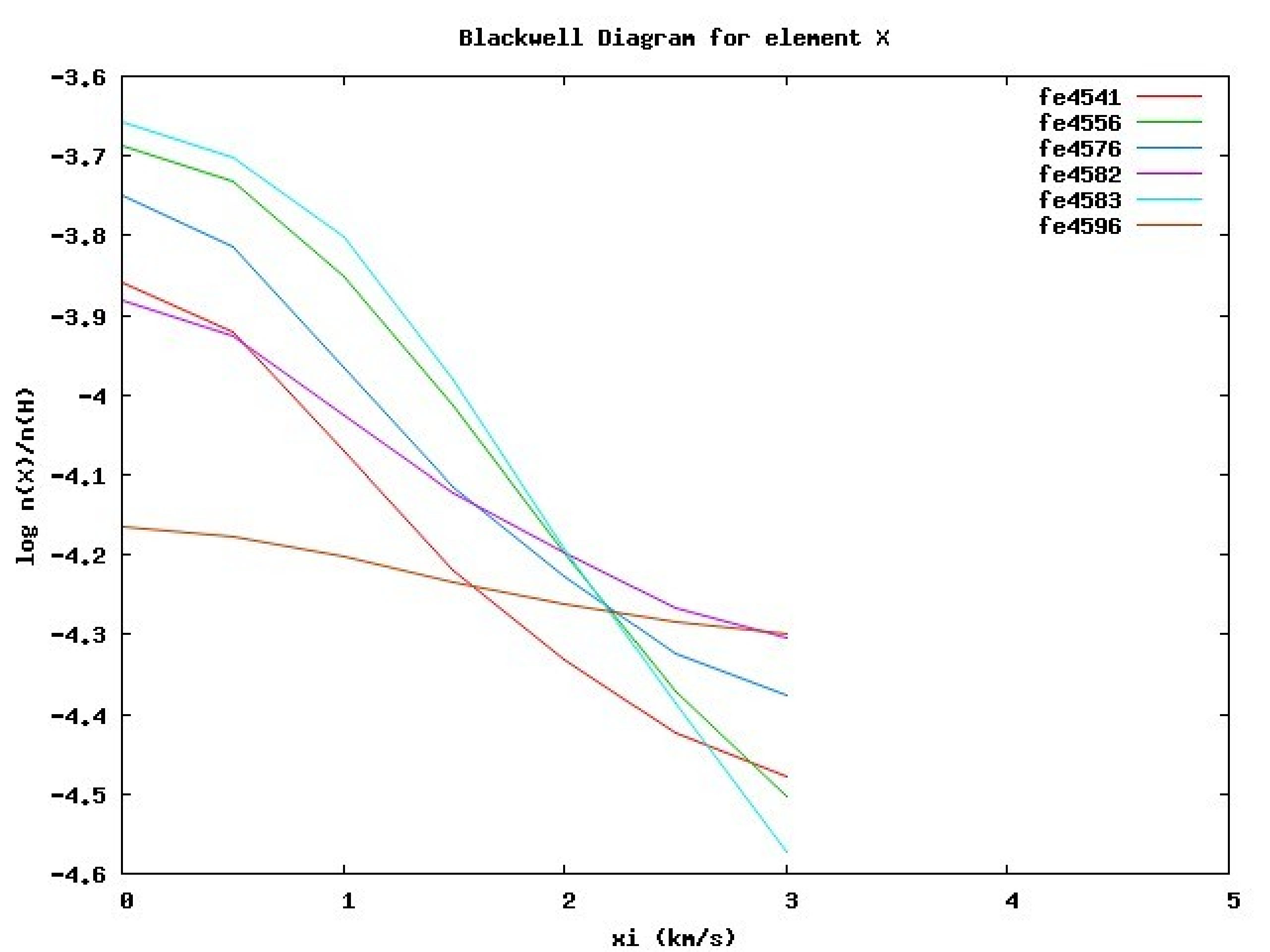
Why is there microturbulence?

- This effect appears to have some physical justification: in stars with strong microturbulence, Schwarzschild convection criterion implies presence of convection
- *This is a very important parameter in abundance analysis, and value determined or assumed has large effect on resulting abundance*
- Difficult to determine at large v sin i because of absence of clean weak lines



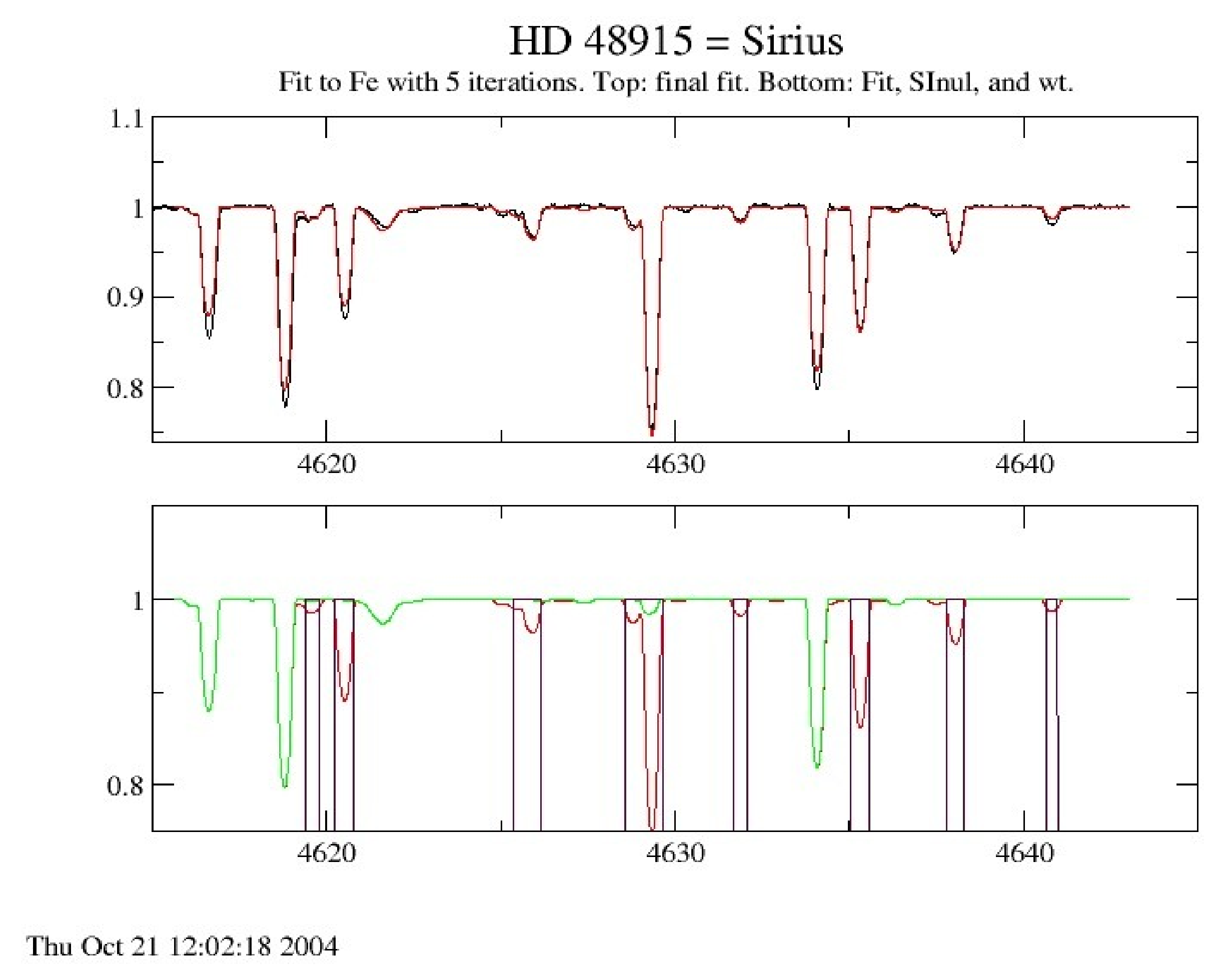
Blackwell diagrams

- One can run Zeeman on several *single* lines for several values of ξ, deduce best abundance for each line and ξ
- Plot of abundance as fn of ξ for all lines (Blackwell diagram) allows one to set abundance, ξ, and to identify problem lines



Global fitting

- Instead of fitting single lines, one can use Zeeman to fit whole spectrum of single element iteratively
- If this is done for several microturbulence values, one can determine best microturbulence parameter as well as deduce abundance
- Different elements -> slightly different ξs...



HD 48915 = Sirius

Fit to Fe with 5 iterations. Top: final fit. Bottom: Fit, SInul, and wt.

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Polarised radiative transfer

- Doing polarised radiative transfer increases computation burden enormously:
  - Each line has numerous Zeeman components, absorption profile of each must be computed at many depths, frequencies, perhaps several rotational phases
  - Must solve four equations of transfer, not one
  - Must do so for many places on star, since field is not expected to be uniform over disk! (In normal spectrum synthesis code, just compute a few angles)
- One of major features of code is polarised radiative transfer, so we need to discuss this
- Discussion in same style as my first lecture about radiative transfer and abundance analysis – simplest possible case

The Stokes vector

- To described *polarised* light mathematically we use the Stokes parameter description: $[I, Q, U, V]$
- I is the total intensity of light in the beam
- For Q and U, measure the intensity of the beam through perfect linear polarisers (polarising analysers) orientated at 0, 45, 90, and 135 degrees. $Q = I_0 - I_{90}$, $U = I_{45} - I_{135}$.
- Measure the intensity of the beam thorough two perfect circular polarisers. $V = I_{right} - I_{left}$.
- These four quantities adequately describe the polarisation state of a light beam
- $[I, Q, U, V]$ are functions of frequency and direction
- Q, U, V are sometimes normalised to I ($Q \to Q/I$, etc).

Opacity ratios in ERTs

- Define η_p, η_l, η_r as the ratios of total (Voigt line profiles + continuum) opacity, in pi, right sigma, and left sigma Zeeman components to the continuum opacity. Then

  $$\eta_I = 0.5\,\eta_p \sin^2\psi + 0.25\left(\eta_l + \eta_r\right)\left(1 + \cos^2\psi\right)$$

  $$\eta_Q = 0.5\,\eta_p - 0.25\left(\eta_l + \eta_r\right)\sin^2\psi$$

  $$\eta_V = \left(\eta_r - \eta_l\right)\cos\psi$$

  Ray of interest defines vertical, angle 0 of linear polarisation is plane of field and LOS, and ψ is the angle between field and vertical.
- Similar expressions involving Faraday-Voigt function are required for the retardance terms
- Note that η_Q and η_V are differences of Zeeman opacity coefficients, like *Q* and *U*. They act to introduce polarisation.

Equation of radiative transfer polarised radiation

$$\mu\frac{dI}{d\tau_\nu}=\eta_I(I-B_\nu)+\eta_Q Q+\eta_V V$$

$$\mu\frac{dQ}{d\tau_\nu}=\eta_Q(I-B_\nu)+\eta_I Q-\rho_R U$$

$$\mu\frac{dU}{d\tau_\nu}=\rho_R Q+\eta_I U-\rho_W V$$

$$\mu\frac{dV}{d\tau_\nu}=\eta_V(I-B_\nu)+\rho_W U+\eta_I V$$

- For polarised radiative transfer, ERTs are written in terms of Stokes parameters
- First order linear DEs like 1D ERT, but now four coupled equations, one for each Stokes parameter
- No scattering is included
- In LTE, B_ν is Planck function
- Factors η describe absorption, factors ρ describe retardation (anomalous dispersion)

Comments on polarised ERTs

- For given atmospheric structure, we can start the 4 polarised ERTs with unpolarised black-body radiation at great depth, follow many rays (many surface grid points, many frequencies) to surface and compute (discrete approximation of) emergent flux spectrum
- Equations describe effects of polarised absorption and retardation on outflowing radiation
- For non-degenerate stars, polarisation is introduced essentially by polarised Zeeman line components
- Cannot accurately predict intensity or polarisation of emergent stellar spectrum without solving these equations – even intensity spectrum is substantially different from what is computed with a single unpolarised equation of transfer
- The point: abundances in magnetic stars are not accurate if deduced with normal spectrum synthesis

Analytical solutions

- As for the unpolarised equation of transport, there is an analytic solution to the simplest case of the polarised equations assuming a linear source function and a normal Zeeman triplet (worth playing with to build intuition)
- Simplest form was derived by Unno (1956, PASJ 8, 108) in paper which laid the basis for setting up ER
- T in terms of Stokes parameters. Note that this paper ignores retardation.

$$\frac{I_0(0,\theta)-I(0.\theta)}{I_0(0,\theta)}=\frac{\beta\cos\theta}{1+\beta\cos\theta}\left[1-\frac{1+\eta_I}{(1+\eta_I)^2-\eta_Q^2-\eta_V^2}\right]$$

$$\frac{V(0,\theta)}{I_0}(0,\theta)=\frac{-\beta\cos\theta}{1+\beta\cos\theta}\left[\frac{\eta_V}{(1+\eta_I)^2-\eta_Q^2-\eta_V^2}\right]$$

- See also Martin & Wickramasinghe 1979, MNRAS 189, 883

Available atmosphere codes & models

- Model atmospheres may be computed using either unpolarised or polarised transfer.
- Until recently, only unpolarised model atmospheres were available
  - Hot stars: ATLAS (cf http://kurucz.harvard.edu/ , especially grids and programs)
  - Cool stars: MARCS models (B Gustafsson et al; see also http://www.uwosh.edu/mike/exercises/marcs/marcs.html)
  - Wide range: Phoenix models (P Hauschildt et al)
- D Shulyak and S Khan have recently developed LLModels, a code for hot stars that can compute models including magnetic polarisation effects and arbitrary abundance tables
- It mostly seems to be an acceptable approximation to use unpolarised model atmospheres, or ones with simple line splitting, but the appropriate abundance table has important effects.

Computing an emergent spectrum: computational demands

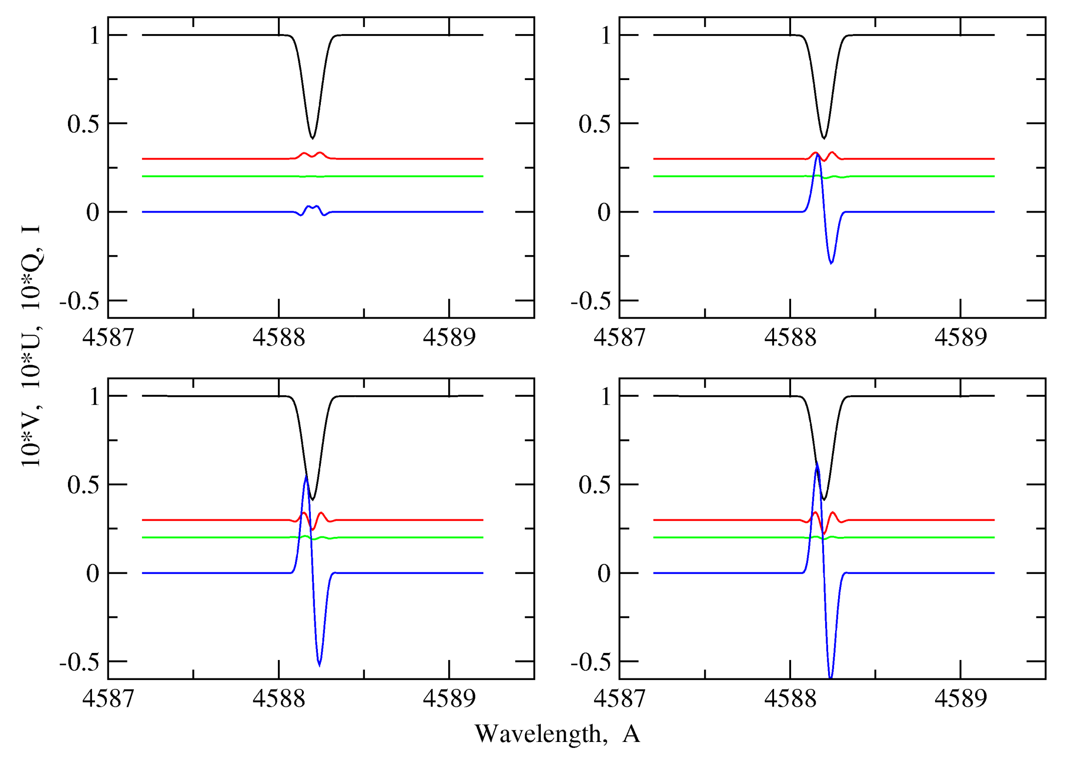
- Given a suitable model atmosphere (T_{eff}, log g, abundances), what do we have to do to compute the emergent polarised spectrum of a magnetic star (forward computation)?
  - Assume some magnetic field structure, and calculate the vector field at many (50+) grid points on the visible hemisphere
  - For each grid point compute detailed run of polarised opacities and retardances at all relevant depths (60+) at closely spaced frequencies or wavelengths (0.01 Å is barely adequate wavelength grid in visible). (Each line has several Zeeman components.) A window of 100 Å might be a useful size.
  - Then compute the emergent spectrum along the ray towards observer at each surface grid point, by solving the ERTs outward along the ray, for each wavelength in the window
- A lot of bookkeeping is required!

Computing the emergent spectrum: techniques

- Several aspects of this problem require special considerations
  - Because one must compute the Voigt and Faraday-Voigt functions millions of times, an efficient algorithm is needed
  - Solving the equations of transfer numerically may be done with standard packages or methods (e.g. Runge-Kutta), but again this has to be done so many times efficiency is essential, and you want a technique that does not require a very dense depth grid
  - In Zeeman, solution is done by using analytical solution to propagate outward from one grid level to next. For the case involving anomalous dispersion, lots of nasty bear traps arise wherever one or more coupling coefficient goes to zero, so about a dozen special solutions must be used.

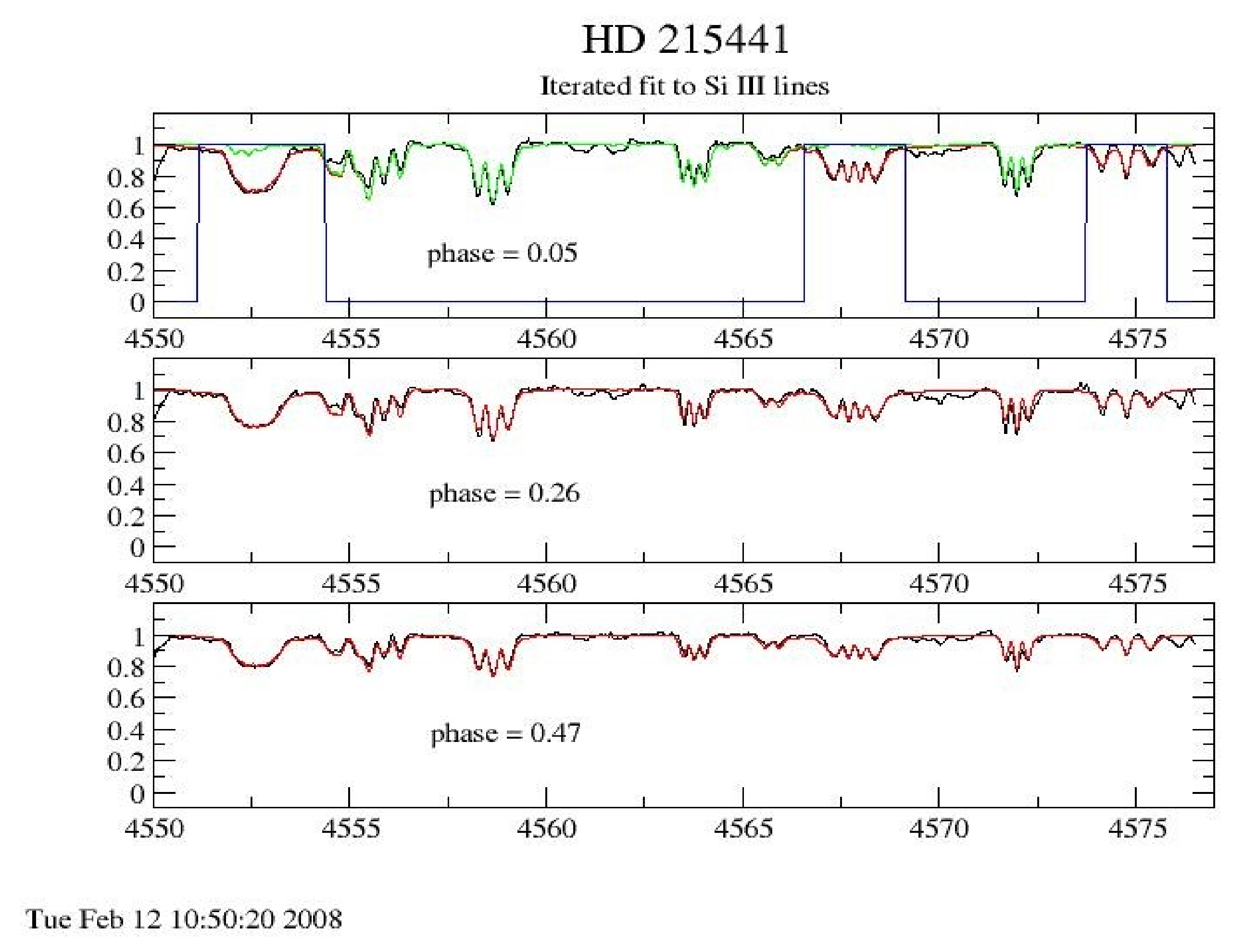
Examples of magnetic line profiles

- Example of line synthesis
  - Cr II 4588 in A0 star
  - Dipole field, polar field strength 1000 G (0.1 T)
  - Star not rotating
  - View from four inclinations from magnetic pole: 0, 30 60, 90 degrees
  - *Q*, *U*, *V* all multiplied by 10
- Note how much larger *V* is than *Q* or *U*

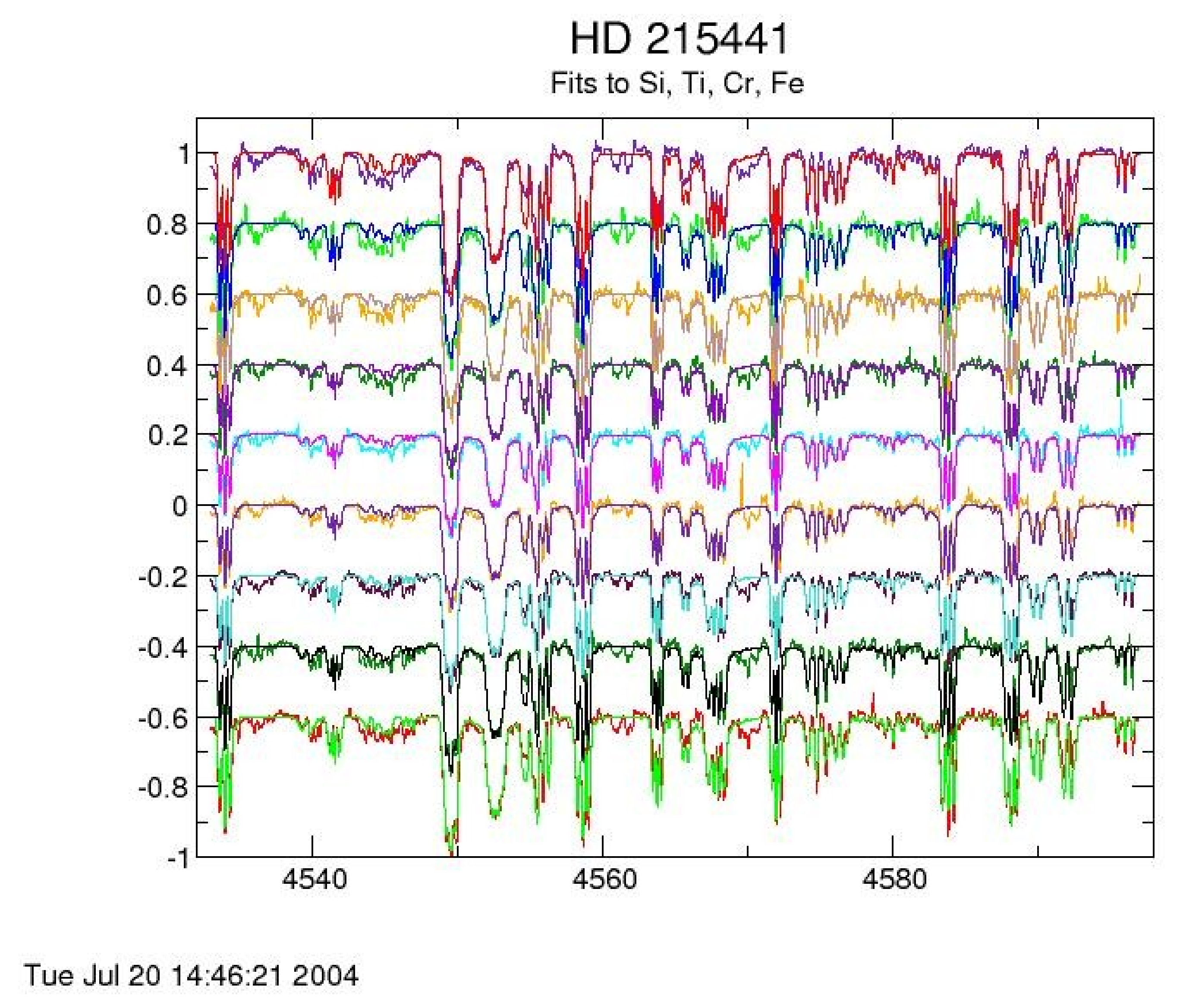


Fit to magnetic Ap stars

- Zeeman can iterate abundance for one element in several spectra at once, with different values on each of six rings
- Example for Si III in Babcock's star HD 215441
- Notice that line fits are not very accurate – basic field and abundance models are not maps but models

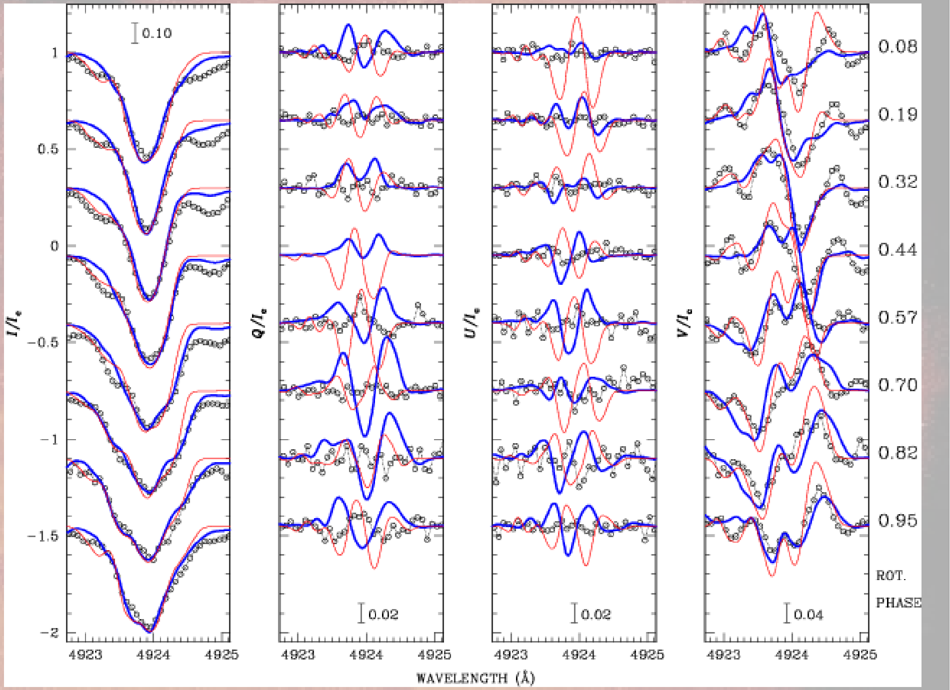


Fit to full rotation cycle of HD 215441



Modelling a magnetic star: tests of parametrised models

- Using MuSiCoS (or ESPaDOnS) data, we can test models derived from I spectra and simple field measurements ($\langle B_z \rangle$ or other field moments), by observing *[I, Q, U, V]* spectra as a function of rotational phase and comparing to computed line profiles based on the model derived from moments (e.g. 53 Cam)
- Result: poor fits, especially to *Q*, *U*.... Simple field models from field average measurements are only first approximations to real structure. This may often be useful but is less than a map.



Would you like to try Zeeman?

- A copy of the source code of Zeeman is available, along with a description of how to get started, some sample data, and some exercises you can try to get familiar with various features, at http://www.astro.uwo.ca/~jlandstr/armagh/
- This directory will also include pdf copies of the talks I am giving at this workshop.
- The zeeman4.tgz file is intended to make it possible for you to use Zeeman, at first for simple applications but later for more difficult things, if you desire. Please feel free to try it out.