A Gentle Theoretical Introduction to Abundance Analysis

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Abundance analysis

- In 19th century it was common knowledge that we would never know chemistry of the distant stars
- The fact that we *can do remote abundance analysis* is one of the most astonishing discoveries of 20th century
- One of earliest results of quantitative spectroscopic analysis, by Cecilia Payne at Harvard (PhD 1925 – one of the most brilliant PhD theses of the 20th c) was the discovery that the stars are made almost entirely of *hydrogen and helium*, not of the common materials of earth
- Seemed so improbable that Payne thought that this result could not be right ... but it is ...
- And cosmology has never gotten over this discovery

What makes abundance analysis possible?

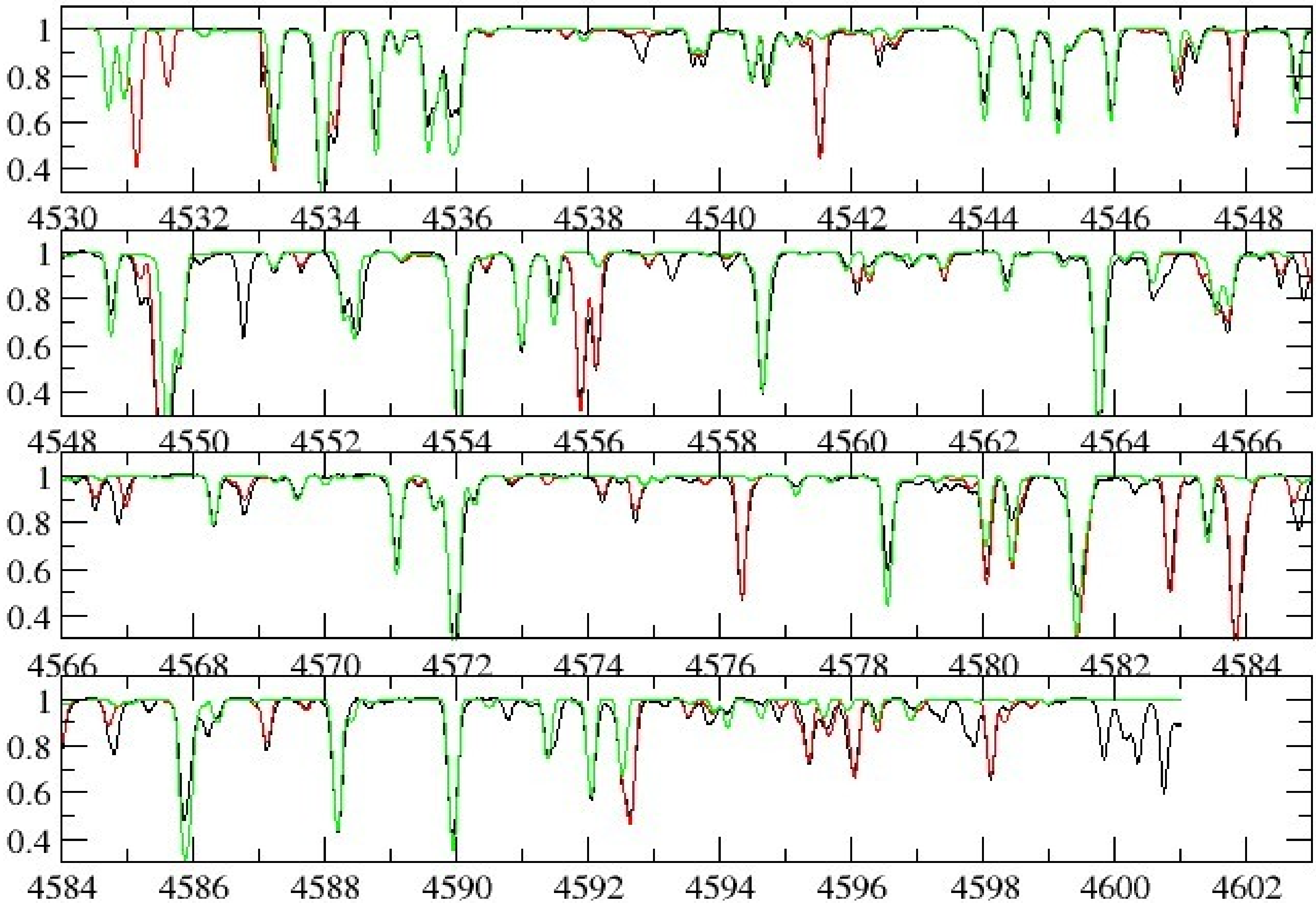
- We have massive objects which have released enough gravitational energy (a) to radiate at ~10^4 K and (b) to ignite nuclear reactions to keep this going for 10^{10} yr
- Atoms have discrete energy levels between which transitions lead to absorption & emission of discrete frequencies – hence atomic species can be *identified*
- Populated states, and even degree of ionisation, depend on temperature, so temperature and pressure in stellar photosphere can be determined
- Radiation leaving star is imprinted with signatures of atoms present and of state of radiating plasma
- Our task is to use all this qualitatively for *chemical and physical analysis of outer layers of star*

Why build models of spectra??

- Some information is available “directly” from spectrum (e.g. radial velocity, v sin i, presence of Fe or Eu, hemispheric average magnetic field strength).
- “Geometric” models can often be made to fit such data, for example the simple picture of a spectroscopic binary, or the limb-darkened simple rotator fitting v sin i data
- Beyond such pictures, synthetic spectra are needed
  - For quantitative information, such as abundance of various chemical species, or distribution of magnetic field over surface
  - To test ideas about physical conditions at surface of (or near) an unresolved star – stratification, velocity fields, etc
- Basic idea: predict *line spectrum* corresponding to hypothesis, and compare prediction with observed spectrum.
- This process is iterated to convergence – if possible!
- Discrepancies provide hints to physics missing from model

HD 61421 = Procyon: F5 IV-V

Identification of clean lines of Fe (in red)



Basic process of (forward) spectrum modelling

- Two essential components are required:
  - fluid structure of outer layers of star from which radiation can escape (a “model atmosphere” - *T(z)*, *p(z)*, etc)
  - associated radiation field, usually described by “specific intensity” $I_\nu(z,\theta)$ of radiation (units: ergs/s-cm^2-Hz-sterad), which is *not* simple isotropic black-body radiation field of a gas at constant temperature because of *leakage*
- Let's briefly remind ourselves of the basic nature of the environment we want to model – where do the various constraints on the system come from, and what qualitative understanding do they offer?

The model atmosphere - basics

- From unchanging spectra of stars and of (integrated light from) Sun, we infer that we may assume hydrostatic equilibrium, for atmosphere & whole star
- From known mass and radius of Sun we can estimate central temperature $T\sim$ few MK required for support by gas pressure (NB radius must be known to do this)
- From energy distribution of sun/starlight, we find radiating layers at $T\sim$ few kK
- Deduce that there is a temperature gradient outwards in atmosphere, in agreement with observation that a *lot* of heat flows out of stars
- So atmosphere will be in hydrostatic equilibrium but have a temperature gradient, hence not in strict thermal equilibrium

More atmosphere basics

- Atmosphere, the region from which we get radiation, must be fairly transparent – optical depth only about 1
- Hence there cannot be a huge temperature gradient – maybe a factor 2 change in T across visible layers
- As rough approx, assume T is constant. Then hydrostatic equil => $p = p_0 \exp(-z/H)$, where H is scale height, $H = kT/(\mu m_A g)$ where μm_A is mean atomic mass and g is acceleration of gravity. Typically $\mu \sim 1$ and H has a value of few hundred km in main sequence star
- Observed atmosphere will have larger density & pressure contrast than temperature contrast, but even if important atmosphere is several scale heights deep, it is thin compared to radius, and therefore “flat” -> 1D

More atmosphere basics

- We have hydrostatic equilibrium: $dp/dz = -\rho g$ to work with, and the gas law $p = \rho kT/\mu m_A$, so we have 4 variables but 2 equations. We need one more constraint to get *T(z), p(z),* etc.
- (We can get an *approximate* atmosphere by assuming that *T(z) = constant*, the atmosphere with exponential drop-off of pressure and density, but this does not predict an absorption line spectrum! Not good enough.)
- We notice again that stellar atmospheres are often steady. So they don't accumulate energy or lose it. This suggests that we should use *integrated flux = constant (z)*, so that no energy is deposited or sourced anywhere in the atmosphere, but simply flows steadily through: what comes in at the bottom goes out at the top

Stellar atmospheres: the moral

- But *integrated flux = constant(z)* is a requirement on the radiation field, not on the temperature. So we must look at radiative transfer to figure out how to translate a requirement on the radiation field into one on *T(z)* : we can't even get the model atmosphere without it.
- We also need to deal with radiation field to be able to predict the spectrum emitted by the gas in the atmosphere
- So we need to examine the **Equation of radiative transfer (ERT)**
- The equation of radiative transfer is the most complicated and obscure aspect of physics of stellar atmospheres. We will try to shed a little light on it....

Equation of radiative transfer: a (very) simple introduction

- We start by looking at a very simple form of this equation, to build intuition, and then look at some of the physics that enters, and (later) look at polarised radiative transfer
- Basic idea is to look at single rays of radiation traveling through the gas, and see how this is absorbed and emitted
- To describe this radiation field, the fundamental quantity is the specific intensity $I_\nu(z, \omega)$ of radiation passing through small volume of gas in a specific direction. We compute changes due to absorption and emission:

$$dI_\nu = \left(-\kappa_\nu I_\nu + j_\nu\right) ds$$

where κ_ν is the absorption coefficient (cm^2/cm^3; i.e. fractional absorption per cm) and j_ν is the emissivity per unit volume (ergs/s-cm^3-Hz-sterad)

ERT: simple introduction (2)

- Consider a stellar atmosphere with *ds* at an angle θ to the vertical, so $ds = dz/\cos\theta = dz/\mu$. Now define (monochromatic) optical depth $d\tau_\nu = -\kappa_\nu\, dz$, which measures fractional extinction along a path. Then

$$\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - \frac{j_\nu}{\kappa_\nu} = I_\nu - S_\nu$$

  This is the equation of transfer (ERT).
- Some important points
  - This form ignores all scattering processes
  - In this approximation, the ERT describes the variation of radiation along a single ray. Each ray is *independent* of all others, and each has its own ERT
  - If we start a ray deep inside an object and follow it to the surface, this equation predicts the emerging light $I_\nu(0, \omega)$

Solving our simple EOT

- In "local thermodynamic equilibrium" (LTE), S_ν can be approximated as the Planck function $B_\nu(T)$.
- If we multiply the equation of transfer by the integrating factor exp$(-\tau_\nu/\mu)$, and assume that S_ν is a known function of τ_ν, the ERT can be integrated.

$$I_\nu(\tau_\nu) = e^{\tau_\nu/\mu} \int_{\tau_\nu}^{\infty} S_\nu(\tau_\nu') e^{-\tau_\nu'/\mu} d\tau_\nu'/\mu$$

- With lower limit 0, we get specific intensity emerging from surface.
- Now suppose that $S_\nu(\tau_c) = S_0(1 + \beta\tau_c)$, for τ_c of a particular (continuum) frequency and nearby frequencies (simplest approximation). Integrating:

$$I_c(0) = S_0(1+\beta\mu) = S_c(\tau_c = \mu)$$

ERT: finally, line absorption

- Also suppose that $\kappa_\nu = \kappa_c + \kappa_{line} = (1+\eta_\nu)\kappa_c$
  where η_ν is a function of frequency but not depth.
- Then $\tau_\nu = (1+\eta_\nu)\tau_c$ and $S_\nu(\tau_\nu) = S_0[1+\beta\mu\tau_\nu/(1+\eta_\nu)]$
  so finally we get an expression for surface intensity
  $$I_\nu(0) = S_0[1+\beta\mu/(1+\eta_\nu)]$$
- This shows that if we know how to write the line absorption coefficient (normally as a Voigt profile), then we see that as we get near line centre the emergent intensity drops, while in the nearby continuum it has the continuum value found above.
- The point: in this particular case the ERT is a solvable first order, linear DE with a driving term. (Of course, there are a lot of more complex situations!)

Opacities

- We see that even in this simple approximate treatment of line absorption we need the quantity $\eta_\nu = \kappa_\nu/\kappa_c$, the ratio of line to continuum absorption coefficients.
- It's clear that we need the frequency variation of the line absorption coefficient to describe a line, but ...
- why the continuum opacity? It enters through our approximate linear source function $S_\nu(\tau_c) = S_0(1 + \beta\tau_c)$, written in terms of the optical depth in the local continuum, which is the physical depth scale for the radiation. In a more realistic atmosphere, we find that $T(\tau_c)$ (but not $T(z)$) takes a (roughly) characteristic form as a result of flux constancy
- Alternatively, we could say that we need the ratio κ_ν/κ_c because the line absorption must be stronger than continuum absorption to change monochormatic brightness much

Line absorption coefficient profile

- We need two kinds of information about line opacity profiles: shape and strength.
  - The shape is usually generic: a Voigt profile, which has an exponential core due to thermal broadening, and weak but broad wings due to collision broadening. There are a number of clever routines to compute this profile, given the local temperature and the ratio *a* of damping to Doppler width.
  - In our approximate problem, we could take this to be a Gaussian $\exp\{-[(\nu-\nu_0)/\Delta\nu_D]^2\}$ where ν_0 is the line frequency and $\Delta\nu_D = (2kT/m)^{1/2}(\nu_0/c)$
  - The damping wings generally become visible only for rather strong lines

Line absorption strength

- The line-to-continuum opacity ratio which is needed in reality is different for each level in the atmosphere and is proportional to
  - The *f* value of the transition
  - The population (number density) of ions in the lower level
  - The value of the Voigt profile at that point in the profile
- And inversely proportional to the local continuous opacity coefficient (its value at the frequency in question).
- In our very simple model, this just appears as a simple numerical parameter η_ν which has no depth dependence (very rough approximation) but does have frequency dependence.

The synthetic spectrum

- Now we return to our Milne-Eddington result,
  $$I_\nu(0) = S_0\left[1+\beta\mu/(1+\eta_\nu)\right]$$
  Clearly, outside our line *(*η_ν *= 0)* the local specific intensity is constant. If the line form is a Gaussian, as one goes through the line in *v*, *I* drops and then recovers, so we have a **very simplified** solution to the ERT that nevertheless leads to spectral lines of about the right shape, with depth depending on the atmosphere model (β) and surface location (μ) as well as on the intrinsic line profile
- To use this result (or such a result) for spectrum synthesis, we must integrate (sum) over the visible stellar hemisphere, usually with Doppler shifts for local rotation velocity, since we see flux, not specific intensity

Relation to abundance analysis

- The solution obtained here is simply a result that is useful for developing insight, not for real spectrum synthesis (but it is certainly worth playing with for a while!)
- However, it shows schematically how, starting from ERT, one can arrive at model spectral lines which could be compared with observations and used to deduce model parameters: a simple abundance analysis
- Even with this model we could carry out abundance analysis by adjusting local line model parameters β, μ (model atmosphere parameters) and η (abundance parameter, together with radial velocity and $v \sin i$.

Real spectrum synthesis

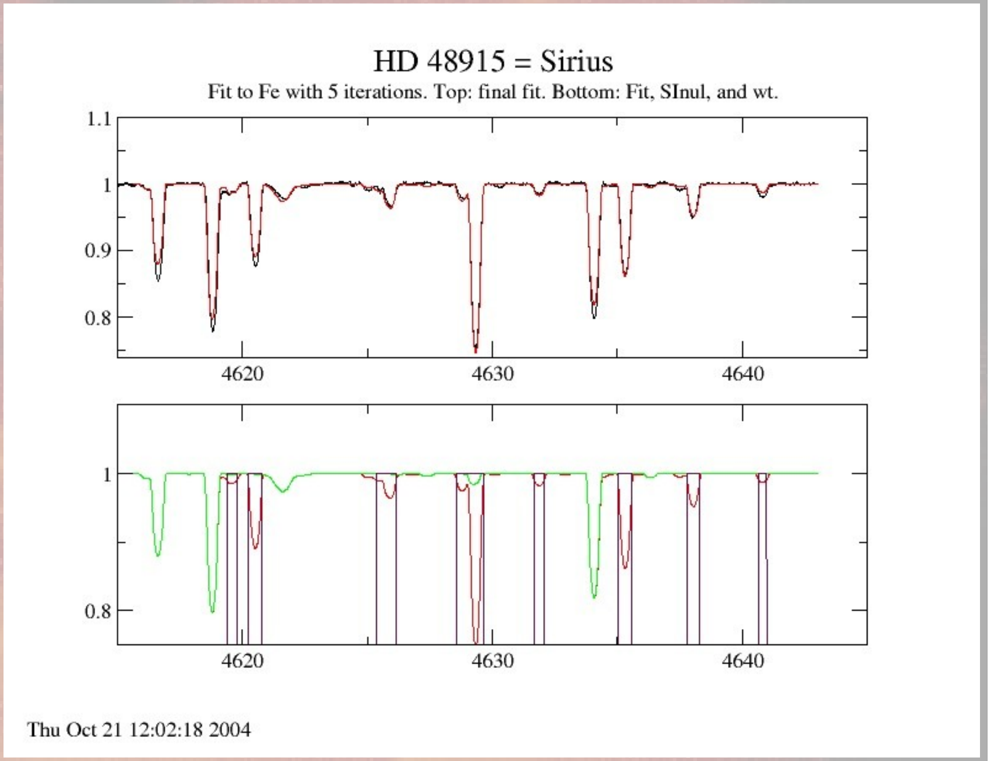
- Our simple model illustrates the logic of spectrum synthesis. What are major differences with real synthesis?
- Much better model atmospheres are available than linear source function
- Basically atmosphere is obtained from calculation sketched earlier:
  - Hydrostatic and thermal equilibrium assumed; 1D
  - *T(z)* assumed, thermodynamic conditions evaluated at each level with *p* and ρ from hydrostatic equilibrium
  - Opacity computed at all wavelengths at each level (60 – 300 levels) of atmosphere, including both continua and (maybe millions of) lines
  - ERT solved for many frequencies and directions. Flux at each level evaluated. At first it is not constant with *z.*
  - *T(z)* is corrected, process repeated until flux is constant

Finally, spectrum synthesis and abundance analysis

- With suitable model atmosphere(s) available, compute detailed spectrum for comparison with observations
- At each level must evaluate thermodynamic conditions to find ionisation and excitation states of ions of interest
- For each transition, need *gf* values and damping constants to calculate local line absorption profile per atom
- At each level must compute total continuum opacity through region of interest to evaluate line/continuum ratio
- Then must solve ERT for many rays and frequencies to get emergent intensity at each point on stellar disk
- Finally, integrate over visible hemisphere (with Doppler shifts as needed) and compare with observed spectrum

Modelling a normal star: finally – abundance analysis

- Synthesis fits to *non-magnetic* stars may be very accurate
- Require good choices of T_{eff}, log g, abundances, radial velocity, v sin i, and microturbulence parameter
- T_{eff} and log g often chosen from available Stromgren or Geneva photometry calibrations
- Automated iterative fitting of most remaining parameters works well for such stars



A final example

- An example of a fit to a fairly sharp-line Am star
- Notice large number of parameters that are determined from 30 A spectrum plus photometry
- Be careful specifying abundance – relative to what? n_X/n_{tot}? n_X/n_H?
- If relative to n_{tot}, you must specify He/H at least if not measured