

# Synthetic photometry for non-radial pulsations in subdwarf B stars

B. Ramachandran<sup>1</sup>, C. S. Jeffery<sup>1</sup>, and R. H. D. Townsend<sup>2,3</sup>

<sup>1</sup> Armagh Observatory, College Hill, Armagh BT61 9DG, Northern Ireland, UK  
e-mail: brc@arm.ac.uk

<sup>2</sup> Bartol Research Institute, University of Delaware, Newark, DE 19716, USA

<sup>3</sup> Department of Physics & Astronomy, University College London, Gower Street, London WC1E 6BT, UK

Received 10 May 2004 / Accepted 12 August 2004

**Abstract.** We describe a method for computing theoretical photometric amplitude ratios for a number of modes of nonradially pulsating subdwarf B stars in both SDSS and *UBVR* systems. In order to avoid costly solutions of the non-adiabatic non-radial pulsation equations, we have adopted the adiabatic approximation. We argue that this is a valid approach, at least for the V361 Hya stars, because observations show that the temperature perturbations dominate the radius perturbations in the flux variation. We find that for V361 Hya stars, low-degree ( $\ell = 0, 1, 2$ ) modes may be difficult to distinguish using optical photometry. However, the high degree modes ( $\ell = 3, 4$ ) are relatively well separated and may be distinguished more easily. We have also computed the amplitude ratios for a number of modes in PG 1716+426 stars. For these stars, the amplitude ratios for low degree modes ( $\ell = 0, 1$ ) are well resolved. For oscillations with periods  $\sim 40$  min, higher-degree modes ( $\ell = 2-4$ ) may also be identified easily from their amplitude ratios. However for longer period oscillations, the  $\ell = 3$  and the  $\ell = 2, 4$  modes approach the  $\ell = 0$  and  $\ell = 1$  modes respectively.

**Key words.** stars: subdwarfs – stars: atmospheres – stars: variables: general

## 1. Introduction

Subdwarf B (sdB) stars are identified with the extended horizontal branch stars in the Hertzsprung-Russell diagram. They are low mass stars ( $M \sim 0.5 M_{\odot}$ ) that burn helium in their core and possess an extremely thin hydrogen envelope that is unable to burn hydrogen (Heber 1986). These stars evolve into low mass white dwarfs and are the main contributors to the ultraviolet excess from the giant elliptical galaxies (Brown et al. 2000).

A few years ago, a number of sdB stars were discovered to pulsate (see Kilkeny 2002, and references therein). At the same time independent theoretical calculations showed that, under certain circumstances, pulsations were likely to be excited in extended horizontal branch stars (Charpinet et al. 1996). These stars, now known as V361 Hya stars (originally called EC14026 variables after the prototype EC14026-2647), are multi-periodic pulsators with periods of a few hundred seconds. The pulsations have been identified with low-order and low-degree p modes driven by the opacity ( $\kappa$ -)mechanism, with iron-group element opacities (Z-bump) being responsible for the excitation (Charpinet et al. 1996, 1997).

More recently, another group of pulsating sdB stars has been discovered with periods of around one hour (Green et al. 2003). These are referred to as PG 1716+426 stars after the

prototype of this name<sup>1</sup>. Recent models propose that these pulsations are due to high-degree gravity modes excited by the same Z-bump  $\kappa$ -mechanism as the V361 Hya stars (Fontaine et al. 2003).

The pulsating sdB stars offer an excellent possibility for doing asteroseismology because the frequency distribution is sensitive to the structure of the outer layers of the star. Already efforts have been undertaken in this direction (Brassard et al. 2001).

An asteroseismic analysis requires a precise determination of frequencies and the identification of the modes corresponding to these frequencies. The modes of oscillation are represented by the radial order ( $n$ ), the spherical degree ( $\ell$ ) and the azimuthal number ( $m$ ). For nonrotating stars, the frequency of the mode is independent of  $m$ . Identifying  $\ell$  from frequencies alone is difficult as the frequency spectrum can be complicated. Therefore, methods which use additional information from line profile, radial velocity and colour variations have been developed. Spectroscopic methods which rely on line profile variations have been applied to stars such as  $\beta$  Cephei (e.g. Aerts 1996).

---

<sup>1</sup> There are in fact two subdwarf B stars in the Palomar Green catalogue with right ascension 17h 16m (Green et al. 1986), so the shorter epithet PG 1716 is ambiguous.

The photometric method to detect modes started with the pioneering work of Dziembowski (1977) who derived expressions for the bolometric and radial velocity variations for a non-rotating star pulsating nonradially in a single mode. Balona & Stobie (1979) recast Dziembowski's derivation in a form suitable for comparison directly with the observations. Using colour amplitudes and phase differences, Stamford & Watson (1981) attempted to identify the modes in  $\beta$  Cephei and  $\delta$  Scuti stars. Watson (1988) extended this method by incorporating the variation of limb darkening and applied it to ZZ Ceti, 53 Persei, and Cepheid variables. Garrido et al. (1990) and Garrido (2000) derived a method of mode identification based on phase differences and applied it to  $\delta$  Scuti and  $\gamma$  Doradus variables. This method is appropriate for stars in which both radius and temperature variations affect the overall light curve. Heynderickx et al. (1994) used a photometric mode identification and applied it to hotter stars such as  $\beta$  Cephei, in which phase differences are less important because the temperature effect dominates the radius effect. Various authors have used the amplitude ratio method to study the slowly pulsating B stars (SPB) (Townsend 2002; Dupret et al. 2003). Koen (1998) estimated values of  $\ell$  for a V361 Hya star using the amplitude ratio method of Watson (1988). In this paper we derive new theoretical amplitude ratios useful for identifying the spherical degree of nonradial oscillations in both the V361 Hya and PG 1716+426 stars.

## 2. Method

The amplitude ratio method (Heynderickx et al. 1994) is based on the fact that for nonradially pulsating stars the photometric amplitude is a function of wavelength for a particular value of  $\ell$  and is independent of the azimuthal degree  $m$  and inclination angle  $i$ . This property can be exploited observationally either by use of a dispersive spectrograph, an energy-sensitive detector, or a multi-colour photometer. In the latter case, the relative amplitudes between the various photometric passbands needs to be found. This calculation involves a description of the stellar surface.

We compute the figure of the stellar surface as perturbed by one or more pulsation modes, providing local values for effective temperature  $T_{\text{eff}}$ , surface gravity  $\log g$ , normal surface vector, projected area and velocity for the visible stellar surface. This is convolved with a grid of theoretical intensity spectra in order to compute the apparent flux in the wavelength region observed. Finally, the apparent flux is convolved with an appropriate set of transmission functions to simulate the observed photometric indices.

In order to achieve this, we have combined a number of existing computer programs:

- BRUCE (Townsend 1997) is a code written in FORTRAN77 for nonradially pulsating stars which also rotate. It creates an equilibrium surface grid on the photosphere of the star. The grid is a two dimensional mesh wrapped around the entire surface of the star with the appropriate stellar radius and photospheric physical quantities assigned to each grid point. The grid is then subjected to perturbations in

velocity, temperature, normal and area for an arbitrary number of pulsation modes over a progressive sequence of time steps. The parameters given to BRUCE are the effective temperature,  $\log g$ , the polar radius, equatorial rotational velocity, the oscillation period, the nonradial mode ( $\ell$  and  $m$ ), the inclination angle ( $i$ ) and the amplitude of the radial velocity. Although BRUCE is capable of incorporating nonadiabatic effects, we assume throughout that the pulsation behaves adiabatically; we discuss the rationale and validity of this approximation in Sect. 5.

- STERNE (Jeffery et al. 2001) constructs line-blanketed model atmospheres for early-type stars in local thermodynamic, radiative and hydrostatic equilibrium. A grid of models with  $18\,000\text{ K} \leq T_{\text{eff}} \leq 36\,000\text{ K}$ ,  $4.8 \leq \log g \leq 6.0$ , and with  $n_{\text{He}}/n_{\text{H}} = 0.01$  and solar metallicity was computed for this paper<sup>2</sup>.
- SPECTRUM (Jeffery et al. 2001) computes emergent fluxes and specific intensities for a given model atmosphere, wavelength region, angles and a specified list of atomic absorption lines. Specific intensities ( $I_{\ell\mu}$ ) were computed for six angles ( $\mu = 1, 0.8, 0.6, 0.4, 0.2$  and  $0.01$ ) in the wavelength interval  $2900\text{--}8000\text{ \AA}$  on the above grid of model atmospheres. A microturbulent velocity of  $5\text{ km s}^{-1}$  was adopted for all models. Each spectrum includes approximately 178 000 frequency points and considers the contributions from over 133 000 absorption lines. SPECTRUM does not currently treat the convergence of the Balmer series beyond  $\text{H}_{20}$  ( $3676\text{ \AA}$ ) (Fig. 1), so the wavelength region ( $3646\text{--}3675\text{ \AA}$ ) is incorrectly treated. This region represents less than 10% of the flux in  $u'$ , and temperature and gravity effects on the higher-order Balmer lines will follow the behaviour of lines up to  $\text{H}_{20}$ . It is necessary to include these filters because the largest amplitude variations occur in  $u'/U$  for pulsating sdB stars; the systematic errors introduced by omitting Balmer lines and above  $\text{H}_{20}$  are likely to be small.
- KYLIE (Townsend 1997) combines the output from BRUCE and SPECTRUM to generate a series of time-resolved observer-directed disk-integrated spectra:  $F_{\lambda}(t)$ .

## 3. Photometric amplitudes

For transformation into an astronomical photometric system, these fluxes ( $F_{\lambda}(t)$ ) are convolved with the corresponding transmission functions (see, Fig. 1):

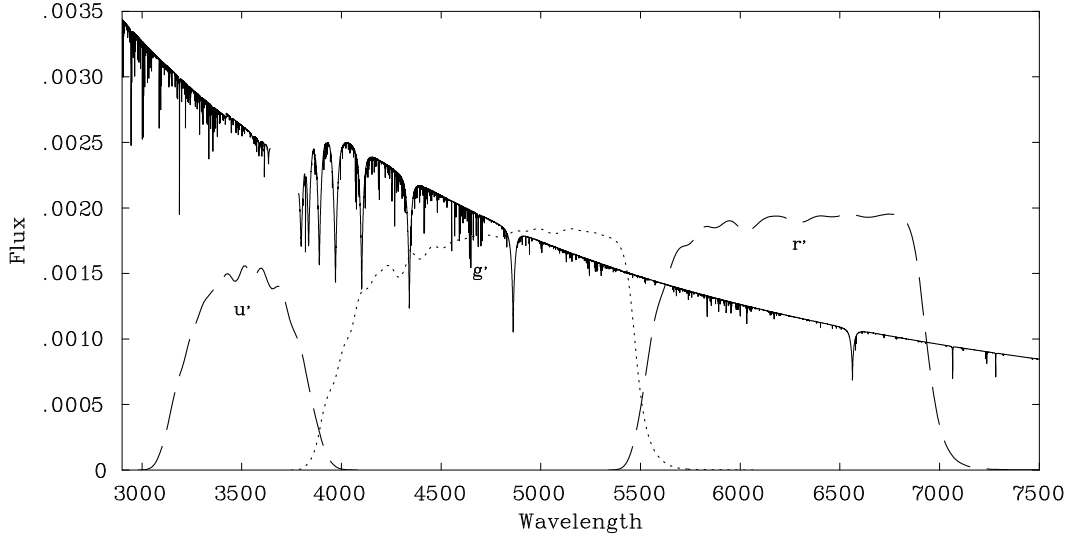
$$F_x(t) = \frac{\int F_{\lambda}(t) T_{x\lambda} d\lambda}{\int T_{x\lambda} d\lambda} \quad (1)$$

where  $T_{x\lambda}$  represents the transmission function and  $F_x$  is the integrated flux in a passband  $x$ .

We convert the variation in  $F_x$  in terms of magnitude by:

$$\Delta m_x(t) = -2.5 \log \left( \frac{F_x^0 + \Delta F_x}{F_x^0} \right) \quad (2)$$

<sup>2</sup> These models are available online at URL: [www.arm.ac.uk/~csj/models/Grid.html](http://www.arm.ac.uk/~csj/models/Grid.html)



**Fig. 1.** A typical sdB spectrum  $T_{\text{eff}} = 32\,000\text{K}$ ,  $\log g = 5.8$  together with the SDSS filter transmission functions (labelled). The gap near the Balmer limit represents a region where hydrogen line opacity is not treated correctly by SPECTRUM.

where  $F_x^0$  is the average flux,  $\Delta m_x$  is the change in magnitude (which is a function of time) and  $\Delta F_x$  is the change in flux. Thus, we simulate the lightcurve in each passband for a number of modes. The amplitude (in magnitude) and phase of the variation in each passband,  $a_x$ ,  $\phi_x$ , can be obtained from the best-fit sine function to  $\Delta m_x(t)$ .

For the present study, we have computed amplitudes in the  $u'$ ,  $g'$  and  $r'$  filters of the Sloan Digital Sky Survey System (SDSS) (Fukugita et al. 1996) as well the  $U$ ,  $B$ ,  $V$  and  $R$  filters of the Johnson-Cousins system ( $UBVR$ ) (Bessel 1990). By convention, we have normalized the amplitudes to the amplitude in the  $u'$  and  $U$  filters in the SDSS and  $UBVR$  systems, respectively, in order to obtain the amplitude ratios.

The theoretical amplitude ratios for each  $\ell$  can then be compared with those obtained from observations. For a given spherical degree  $\ell$ , let the theoretical amplitude ratio  $A_{x,u'}^\ell = a_x/a_{u'}$ , where  $a_x$  is the amplitude in passband  $x$ , and the observed corresponding amplitude ratio be  $(A_{x,u'})_{\text{obs}}$ . Then  $l$  may be constrained, for example, by minimizing the statistic

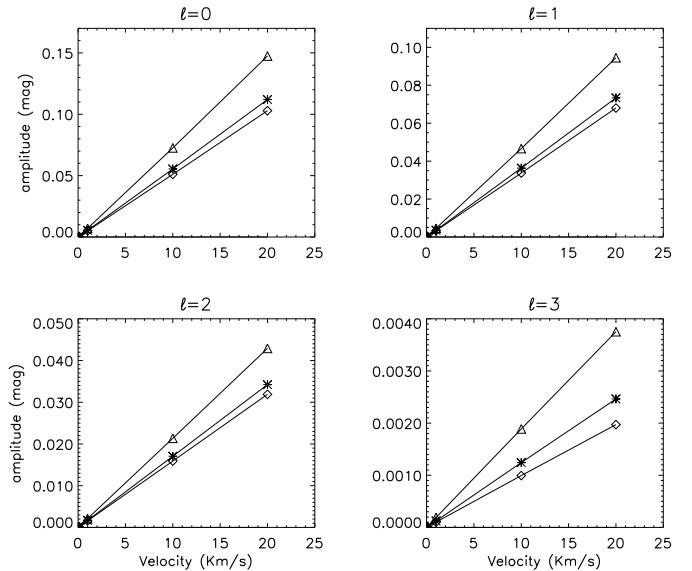
$$D = \sum_x \left[ A_{x,u'}^\ell - (A_{x,u'})_{\text{obs}} \right]^2$$

with respect to  $\ell$ .

#### 4. Analysis

As the sdB stars have a mass of around  $0.5 M_\odot$  (Heber 1986), the polar radius is given directly by the surface gravity. For example, for an V361 Hya star of  $\log g = 5.8$ , the polar radius is  $\sim 0.15 R_\odot$  and for a PG 1716+426 star with  $\log g = 5.5$ , the polar radius is  $\sim 0.21 R_\odot$ . Most of the pulsating sdB stars are slow rotators (Heber et al. 2000, and references therein) and hence we have used an equatorial rotational velocity of  $10 \text{ km s}^{-1}$ .

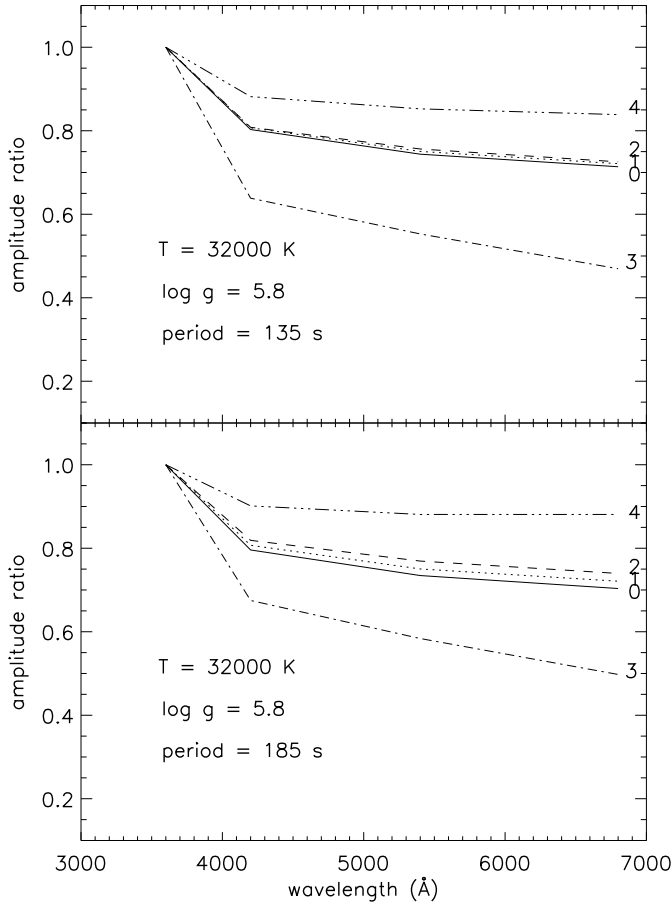
Figure 2 shows the photometric amplitude as a function of velocity amplitudes for various spherical degrees in the three filters of the SDSS system. This figure demonstrates that the



**Fig. 2.** The photometric amplitude (in magnitudes) is plotted with the velocity amplitude for the passbands  $u'$  (triangle),  $g'$  (cross) and  $r'$  (diamond).

amplitudes scale linearly with  $v_{\text{amp}}$  and hence that the *amplitude ratios* are independent of  $v_{\text{amp}}$ . Therefore we have adopted a nominal  $v_{\text{amp}} = 10 \text{ km s}^{-1}$ . Computations were carried out for  $i = 45^\circ$  and  $60^\circ$ , and for all allowed positive values of  $m$ . As expected, amplitude ratios were independent of both  $i$  and  $m$ . In practice, in calculating amplitude ratios, we have usually used  $m$  for which the apparent amplitude is greatest at the adopted inclination.

To model the V361 Hya stars, we have chosen  $T_{\text{eff}} = 32\,000 \text{ K}$ ,  $\log g = 5.8$  and test periods of 135 s and 185 s (cf. Kilkenny 2002). We find that as the period decreases (Figs. 3 and 4), amplitude ratios for  $\ell = 0-2$  lie extremely close to each other at visible wavelengths, while for  $\ell = 3$  and 4, they are well separated. Varying  $T_{\text{eff}}$  by a small amount does not change

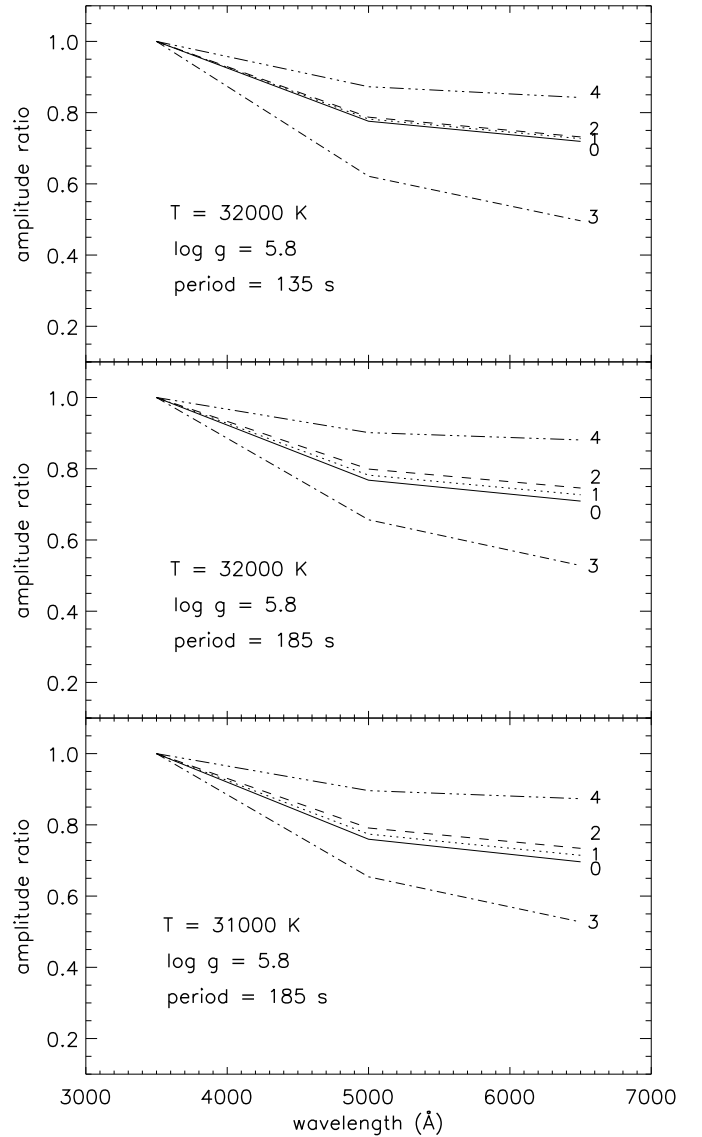


**Fig. 3.** The photometric amplitude ratio as a function of the wavelength for  $\ell = 0$  (solid line), 1 (dotted line), 2 (dashed line), 3 (dashed dot line) and 4 (dashed dot dot line) in a model V361 Hya star of period 135 s and 185 s in the *UBVR* photometric system. We have shown the values of  $\ell$  corresponding to the lines in the diagram.

the plot (third plot of Fig. 4). The small separation of the low degree modes may make it difficult to distinguish modes of low spherical degree, modes with  $\ell = 3$  and 4 should be more easily identified. In fact using these results, Jeffery et al. (2004) have shown that one of the observed frequencies in each of the two V361 Hya stars KPD 2109+4401 and HS 0039+4302 may correspond to an  $\ell = 4$  mode.

To model the PG 1716+426 stars, we have used  $T_{\text{eff}} = 25\,000$  K,  $\log g = 5.5$  and test periods of 40, 60 and 90 min (cf. Green et al. 2003; Fontaine et al. 2003). Figures 5 and 6 show that for a period of 40 min the amplitude ratios for different values of  $\ell$  are well separated. For longer periods, the amplitude ratios for the  $\ell = 0$  (radial) and  $\ell = 1$  modes do not change significantly, but the ratios for higher order ( $\ell > 1$ ) modes move towards higher values so that the  $\ell = 3$  mode is very close to the radial mode and the  $\ell = 2$  and 4 modes are close to the  $\ell = 1$  mode. Therefore, unlike the V361 Hya stars, the low degree ( $\ell = 0$  and 1) modes of PG 1716+426 stars are well separated. Higher-degree modes ( $\ell > 1$ ) may be either well resolved or, depending on the period, difficult to distinguish from low-degree modes.

It should be noted that these results are only valid for slowly rotating pulsating sdB stars. They may not be valid for more



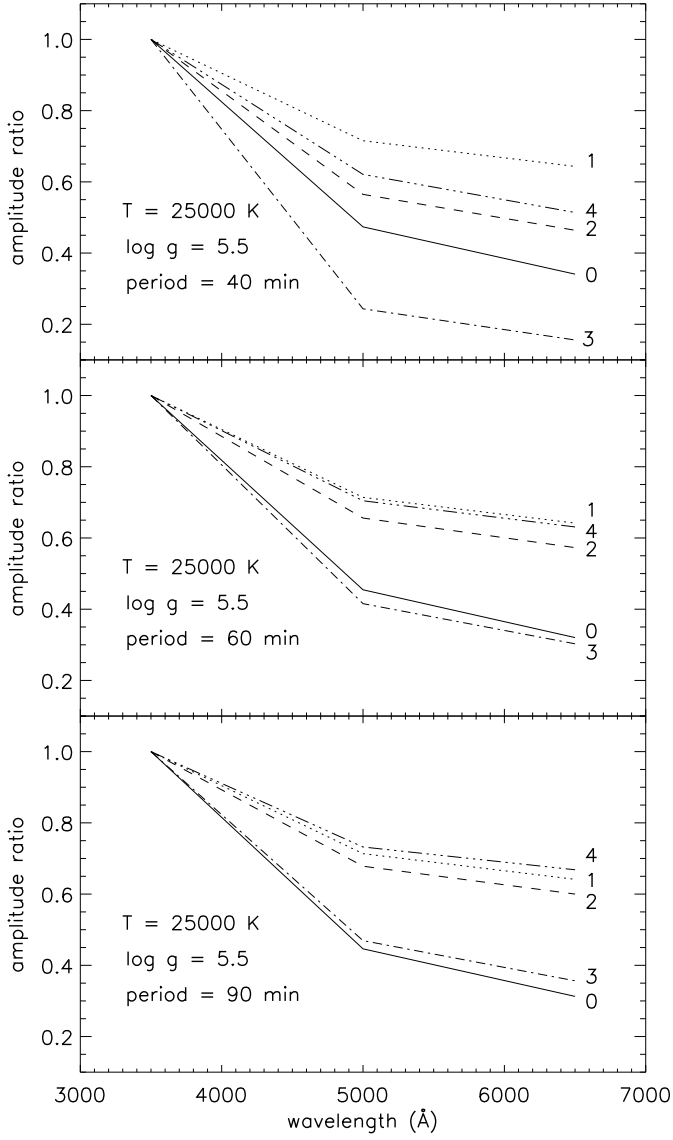
**Fig. 4.** As Fig. 3 in the SDSS system. The bottom panel shows the effect of a small change in  $T_{\text{eff}}$ .

rapidly rotating sdB stars such as PG 1605+072 (Heber et al. 1999).

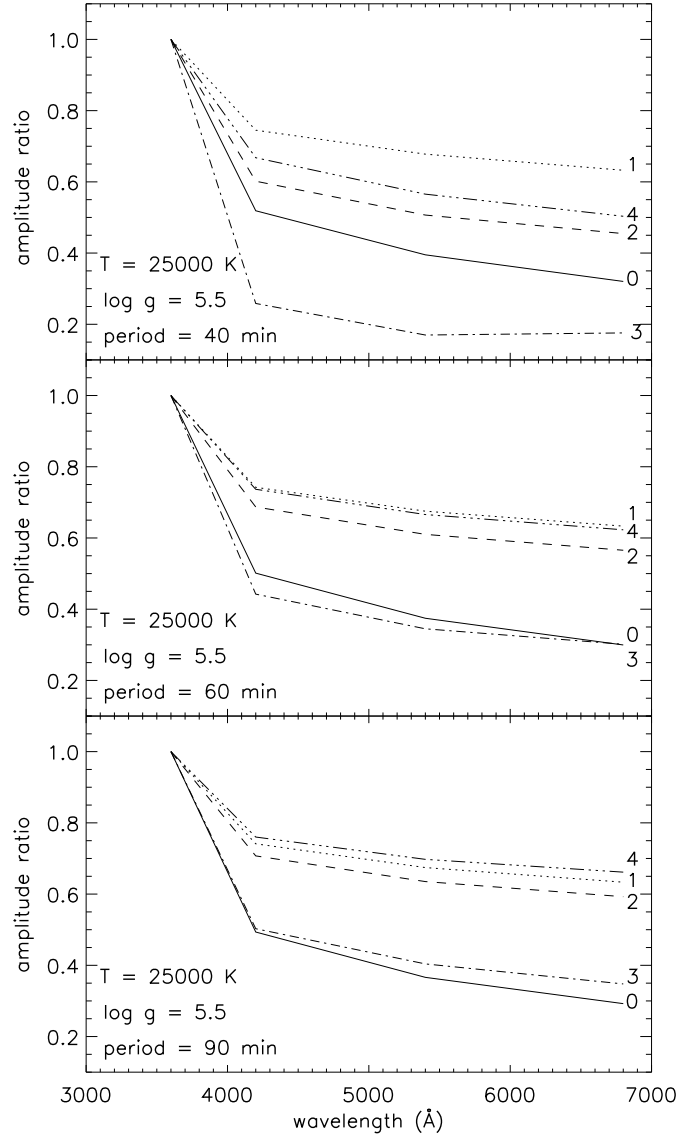
## 5. The adiabatic approximation

Usually the computation of theoretical amplitude ratios involves calculations which require setting up an equilibrium model and solving the pulsation equations in the nonadiabatic limit. However, this is a costly process and can be avoided if the adiabatic approximation can be justified.

To illustrate, equilibrium models of subdwarf B stars have been well studied (e.g., Dorman et al. 1993), but require the inclusion of diffusive processes, including the radiative levitation of iron (Charpinet et al. 2001) and other elements, before pulsational instability can be established. These models have been used successfully as input to full non-radial non-adiabatic pulsation equations in order to reproduce pulsation periods for V361 Hya stars (Charpinet et al. 2001). Initial results for



**Fig. 5.** As Fig. 4 for a model PG 1716+426 star with periods of 40, 60 and 90 min.



**Fig. 6.** As Fig. 5 for a model PG 1716+426 star with periods of 40, 60 and 90 min in the *UBVR* system.

PG 1716+426 stars are also promising (Fontaine et al. 2003). In order to simulate the precise variations in flux and spectrum of these stars, we ought to have used the non-adiabatic amplitudes generated by similar solutions as input to BRUCE. However, this approach can be substituted by the simpler adiabatic approximation *if* the amplitude of the temperature perturbations is very much greater or very much less than that of the radius perturbations.

The flux variations may be represented as (e.g. Townsend 2002)

$$\frac{\Delta F_x}{F_x} = \text{Re} \left[ Y_\ell^m(\theta, \phi) e^{i\sigma t} (\mathcal{T}_x + \mathcal{R}_x) \right] \quad (3)$$

$$\mathcal{T}_x = \frac{T_a}{I_{0,x}} \frac{\partial I_{\ell x}}{\partial \ln T_{\text{eff}}}$$

$$\mathcal{R}_x = \frac{R_a}{I_{0,x}} (2 + \ell)(1 - \ell) I_{\ell x}$$

where  $\Delta F_x$  is the change in the flux in a passband  $x$ ,  $\mathcal{T}_x$  and  $\mathcal{R}_x$  represent the flux variations arising from temperature and radius perturbations respectively,  $T_a$  and  $R_a$  are the complex temperature and radius amplitudes obtained by solving the nonadiabatic pulsation equations,  $Y_\ell^m(\theta, \phi)$  is the usual spherical harmonic,  $\sigma$  is the frequency,  $T_{\text{eff}}$  is the effective temperature and  $\ell$  is the spherical degree.  $I_{\ell x}$  and  $I_{0,x}$  are quantities that depend on the radiation field of the atmosphere.

As observed by Koen (1998) and Jeffery et al. (2004), the light curves in different passbands of V361 Hya stars do not show any phase differences. Moreover, the light curve in the bluest filter has the largest amplitude, indicating that the temperature effect dominates the flux variation. Therefore if we neglect the second term in Eq. (3), the amplitude at a given wavelength depends only on the amplitude of the temperature variations and the radiation field. On taking the amplitude ratio, the contribution is then only from the radiation field term. Hence for V361 Hya stars, amplitude ratios do not depend upon

the amplitudes of the temperature variations or on the radius variations, and our use of the adiabatic approximation will lead to the same results as a fully non-adiabatic analysis. However, in the case of stars where both temperature and the radius effects contribute significantly to the flux variations, a full non-adiabatic treatment is required to compute the amplitude ratios.

For the PG 1716+426 stars, Green et al. (2003) observe that the pulsation amplitude is highest in the bluest filter. There is no published data regarding phase differences. As more multi-colour data are procured, it will become clearer whether these stars behave like their short period counterparts or whether radius effects are more important. For the present, we have investigated the PG 1716+426 star models under the adiabatic approximation. If temperature effects are shown to dominate, our results will be valid.

## 6. Conclusion

We have described a method to compute photometric amplitude ratios due to the non-radial pulsations observed in V361 Hya stars. In order to avoid constructing a detailed interior model and solving the full linear non-adiabatic non-radial pulsation equations, we have adopted the adiabatic approximation. This approach is valid for V361 Hya stars because observations indicate that the flux variation is dominated by the temperature effect.

For V361 Hya stars, the amplitude ratio diagram shows that the higher spherical degrees ( $\ell = 3$  and  $4$ ) should be easily identified using the amplitude ratio method if the observed amplitudes are large enough and the frequencies are sufficiently well resolved. The modes of low spherical degree ( $\ell = 0, 1, 2$ ), however, lie close to one another in the amplitude ratio diagram and will be more difficult to distinguish. This problem becomes more acute for shorter periods.

We have also computed amplitude ratios for PG 1716+426 stars. Modes of low spherical degree ( $\ell < 2$ ) may be easily identified from one another. However, they may be more difficult to distinguish from modes with  $\ell > 1$ , depending on their periods. It remains to be seen whether the temperature variation is dominant for these stars and, hence, whether the adiabatic approximation is fully applicable.

Our calculations have shown that, providing very good multicolour photometry can be obtained, it is possible to identify the spherical degree of some or all of the modes in V361 Hya stars. The same approach may also allow mode identification for PG 1716+426 stars.

*Acknowledgements.* Research at Armagh Observatory is funded by the Department of Culture, Arts and Leisure, Northern Ireland. B.R. is grateful to Sathya Sai Baba for encouragement and also thanks Amir Ahmad and Chia-Hsien Lin for useful discussions. R.H.D.T. is supported by the Particle Physics and Astronomy Research Council of the UK.

## References

- Aerts, C. 1996, *A&A*, 314, 115  
 Balona, L. A., & Stobie, R. S. 1979, *MNRAS*, 187, 217  
 Bessell, M. S. 1990, *PASP*, 102, 1181  
 Brassard, P., Fontaine, G., Billères, M., et al. 2001, *ApJ*, 563, 1013  
 Brown, T. M., Bowers, C. W., Kimble, R. A., Sweigart, A. V., & Ferguson, H. C. 2000, *ApJ*, 532, 308  
 Charpinet, S., Fontaine, G., & Brassard, P. 2001, *PASP*, 113, 775  
 Dorman, B., Rood, R. T., & O'Connell, R. W. 1993, *ApJ*, 419, 596  
 Charpinet, S., Fontaine, G., Brassard, P., et al. 1997, *ApJ*, 483, L123  
 Charpinet, S., Fontaine, G., Brassard, P., & Dorman, B. 1996, *ApJ*, 471, L103  
 Dupret, M.-A., De Ridder, J., De Cat, P., et al. 2003, *A&A*, 398, 677  
 Dziembowski, W. 1977, *Acta Astron.*, 27, 203  
 Fontaine, G., Brassard, P., Charpinet, S., et al. 2003, *ApJ*, 597, 518  
 Fukugita, M., Ichikawa, T., Gunn, J. E., & Doi, M. 1996, *AJ*, 111, 1748  
 Garrido, R. 2000, *Delta Scuti and Related Stars*, *ASP Conf. Ser.*, 210, 67  
 Garrido, R., Garcia-Lobo, E., & Rodriguez, E. 1990, *A&A*, 234, 262  
 Green, R. F., Schmidt, M., & Liebert, J. 1986, *ApJS*, 61, 305  
 Green, E. M., Fontaine, G., Reed, M. D., et al. 2003, *ApJ*, 583, L31  
 Heber, U., Reid, I. N., & Werner, K. 2000, *A&A*, 363, 198  
 Heber, U., Reid, I. N., & Werner, K. 1999, *A&A*, 348, L25  
 Heber, U. 1986, *A&A*, 155, 33  
 Heynderickx, D., Waelkens, C., & Smeyers, P. 1994, *A&AS*, 105, 447  
 Jeffery, C. S., Woolf, V. M., & Pollacco, D. L. 2001, *A&A*, 376, 497  
 Jeffery, C. S., Dhillon, V. S., Marsh, T. R., & Ramachandran, B. 2004, *MNRAS*, 352, 699  
 Kilkeny, D. 2002, *Radial and Nonradial Pulsations as Probes of Stellar Physics*, *IAU Coll.*, 185, *ASP Conf. Ser.*, 259, 356  
 Koen, C. 1998, *MNRAS*, 300, 567  
 Stamford, P. A., & Watson, R. D. 1981, *Ap&SS*, 77, 131  
 Townsend, R. H. D. 2002, *MNRAS*, 330, 855  
 Townsend, R. H. D. 1997, *Ph.D. Thesis*, University College London  
 Watson, R. D. 1988, *Ap&SS*, 140, 255