The Leonids

Leonid Dust Trail Structure and Predictions for 2002

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We discuss the influence of non-linearities in dust trail dynamics caused by the passage of the Earth close to or through dust trails. The effect is to make the derived parameters of these non-linear dust trail sections unreliable for prediction or for use in fitting observed data. These non-linearities become more common in dust trails as they age, but linear sections remain. The timing of encounters with linear sections of dust trails is confirmed as being within 10 minutes and typically ±5 minutes. A qualitative examination of incipient dust trails show that they have a profile that is skewed away from the Sun, that the dust trail profile is a function of Δa_0 and that trails have a dense core at formation which will diffuse out over a few revolutions. Despite this, the density model now gives a reasonable fit over the region of parameter space responsible for storms. There is evidence that the peak region in our model for young trails may be underpredicted due to the existence of this enhanced core. A new model to predict the FWHM of linear dust trail sections is given. The predictions for the two major peaks in 2002 are: (i) 7-rev trail, 2002 November 19, 03:56 ± 5 min UT, ZHR 1000 (810–2000), FWHM ≈ 130 min; (ii) 4-rev trail November 19, 10:34 ± 5 min UT, ZHR 6000 (2900–6000), FWHM 71 min. The 7-rev encounter will have a lower population index than the 4-rev.

1. Introduction

The technique for calculating dust trails from comets has been around for some fifty years, during which time it has been rediscovered on several occasions. Mostly it has been overlooked. It is only in the last four years that clear proof of its predictive value has existed, starting with Reznikov's successful prediction of the 1998 Draconid outburst. The next advance in storm prediction was in 1999 when models for predicting dust trail density, and hence ZHRs, made by McNaught and Asher [1] and Lyytinen [2] were the first to have any success in this regard.

Observations of Leonid outbursts and storms from 1998 to 2001, involving the Earth's passage through numerous dust trails from comet 55P/Temple-Tuttle, has provided a wealth of data on dust trail structure. With this, predictions for 2002 should be considered more reliable than ever before. The only caveat to this is that with this wealth of data, there are now clear indications of the limits of existing models and/or the existence of variations between various dust trails. Inherent variability in the activity of the parent comet is an ever present complicating factor that has had to be ignored in the past due to lack of any specific information to indicate otherwise.

2. The structure of dust trails

In [1] we mentioned the effect of solar radiation pressure (srp) in skewing the distribution of smaller particles to greater $r_\mathrm{E} - r_\mathrm{D}$. Here we investigate this more using a simple ejection model including ejection away from perihelion. Whereas the ejection model may be limited, the general conclusions are robust and these qualitative findings have significant consequences in understanding the structure of a dust trail.

Figure 1 displays six sample cross-sections from ejection in 1899. They refer to the incipient dust trail as it existed in that year and more specifically to the region around $\Delta a_0 = +0.15$ (range $+0.145$ to $+0.155$). We have previously shown that this general structure is basically invariant over several revolutions if gravitation and srp are the only factors considered [1]. We discuss later the effects of other factors which act to diffuse the dust trail structure.

The cross-section in each plot shows a slice through the dust trail in the ecliptic plane with the Sun (or rather the barycenter of the solar system) at $x = 0$, $y = 0$. Due to the inclination of the comet's orbit ($i \approx 162^\circ\!.5$), the cross-section is elongated in ecliptic longitude, but a cross-section perpendicular to the dust trail length is elongated away from the Sun (along the radius vector). Observations show that the elongation in the direction of the radius vector is rather more extended than given in these plots [3].

The top three plots are for $\beta = 0.000$ (no srp) and the bottom three for $\beta = 0.001$ (ratio of srp to solar gravity of 1:1000). The left plots are for an ejection velocity at heliocentric distance $r = 1$ of 10 m/s, middle 25 m/s and right 50 m/s. Ejection at other heliocentric distances is proportional to $1/r$ and ejection occurs $\pm 120°$ in true anomaly ($r \lesssim 3.4$). The plotted points are from a Monte Carlo simulation with the ejection in random directions (uniformly isotropic) and randomly in true anomaly, but only those particles with the specific required Δa_0 are selected.

It is clear from the top left box being blank, (other than the cross representing the comet's node in that year) that for ejection velocities of $10/r$ m/s, particles are unable to reach the required Δa_0. The increased ejection speed of the top middle plot ($v = 25/r$ m/s) results in a compact distribution and the top right plot for even higher velocities is more dispersed. There is a critical low velocity in which ejected particles can just reach the required Δa_0 and such particles are ejected at perihelion (which is close to the node). Just above this limiting velocity, the cross-section is very compact and ejection at true anomalies other than perihelion start to occur. With larger velocities (top right plot), less of the velocity is required to reach the required Δa_0 and more is available to change orbital elements other than the semi-major axis, hence the greater spread in nodes. The distribution is skewed away from the Sun for negative values of Δa_0 and towards the Sun for positive values of Δa_0 although the peak remains around the nominal "center".

The effect of srp is to increase the semi-major axis with respect to a purely gravitational orbit. Thus, in the lower plots with $\beta = 0.001$, the required Δa_0 (with ejection at perihelion) is -0.07, or 0.22 less than the purely gravitational solution. For reasons that need not be discussed here, for non-zero beta, the required value of Δa_0 is variable with true anomaly to have all particles comoving. This is accounted for in the simulations, but the perihelion value of Δa_0 equivalent to $\beta = 0.000$ is always the one quoted.

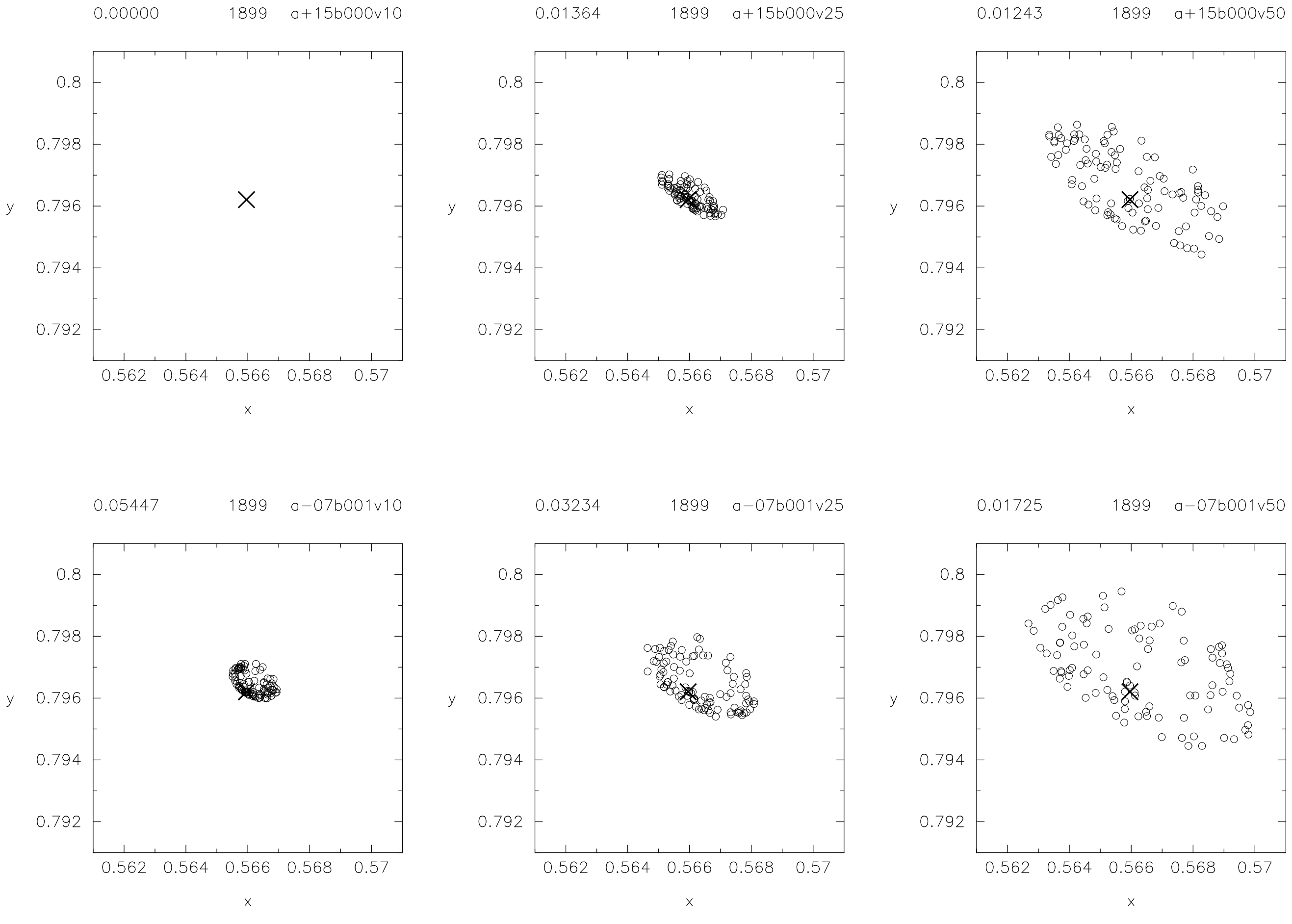

Figure 1 – Monte-Carlo simulations for cross-sections from ejection in 1899.

Whereas the above discussion with $\beta = 0.000$ would apply more to very large particles, for more typical visual meteors, with $\beta = 0.001$, the bottom plots are more representative. Due to the vagaries of the dynamics, it is now possible with non-zero β to have particles which are ejected away from perihelion at some specific v/r to reach the required Δa_0 when ejection at perihelion does not. Overall the whole distribution is pushed to greater heliocentric distance (greater r_{D}).

Combining the full range of velocities and beta will give a smooth dust trail profile without the sharp edges for precise ejection velocities given in the plots. Such a combination would display a prominent core in young trails. Diffusion effects would disperse this central core and change the overall shower profile making it more Gaussian with age. Thus in addition to the profile being a function of Δa_0, it is also a function of age. This may be the reason for our ZHR model underpredicting the near central encounters with the young 1833 (1-rev) and 1966 (2-rev) where diffusion is minimal. This is discussed more in Section 7.

It is of interest that ejection in the solar facing hemisphere alone retains the overall distributions given in these plots. Also, the true anomaly of ejection is correlated with the nodal longitude of the ejected dust. This latter effect can explain activity profile asymmetries due to variations of cometary activity pre and post-perihelion and quasi-periodic variability in the activity curve due to jet activity on a rotating nucleus [4].

3. The density model and its implications

Our 1999 model derived ZHRs from the parameters Δa_0 and $r_{\mathrm{E}} - r_{\mathrm{D}}$ and used a rigorously derived stretching factor f_M as the only diffusion of density with age. An additional aging (diffusion) factor was not included until 2001 due to a prior lack of clear evidence for its necessity.

The nature of these parameters are discussed here.

a) The Δa_0 distribution

Δa_0 represents the necessary change in semi-major axis (which is directly related to orbital period) of meteoroids at the instant of ejection. For an encounter in any particular year, the necessary Δa_0 is that which results in the particles arriving at their node at exactly the same instant as the Earth passes that point. Ejection models of meteoroids from comets imply that more massive (or denser) particles will be ejected at lower velocity and also be less affected by srp. Thus more massive particles tend towards the same orbital period of the comet. Without the action of srp, dust trails would expand symmetrically in front of and behind the comet. The effect of srp is to slightly counteract solar gravity, resulting in particles orbiting in Keplerian orbits more slowly than would otherwise be the case. Smaller particles are thus shifted systematically to higher Δa_0 and for typical visual sized Leonids, this shift is by $\sim +0.2$ (more specifically $+0.22$ for $\beta = 0.001$).

Encounters with a dust trail of a specific age (e.g. the 4-rev) will be at different Δa_0 from one year to the next. In 1998, the relevant section of the 4-rev trail had $\Delta a_0 = +0.04$ and large particles (bright meteors) would have been encountered had the Earth approached close enough to the trail. In subsequent years, the section of the 4-rev trail encountered would have had $\Delta a_0 = +0.08$, $+0.11$, and $+0.14$, the last value being for 2001. From one year to the next, the particles encountered from a specific trail will tend towards smaller sizes. It must be noted that ejection models, including srp, imply a wide range of sizes of particles for any encounter. The population index r will be a function of mass and below a certain mass the numbers of particles can decrease. In calculating r, one should expect it to be magnitude dependent.

It is interesting to note that the 7-rev trail encountered in 2002 will be at $\Delta a_0 = +0.11$, the same as the 4-rev in moonlit conditions in 2000. A similar distribution of magnitudes could be expected were the encounter at the same $r_{\mathrm{E}} - r_{\mathrm{D}}$. With the 7-rev being closer to the core and on the inside of the trail, one would expect brighter meteors as noted in Section 2. It is also probable that the 7-rev trail will involve a lower magnitude distribution than the 4-rev trail this year (with $\Delta a_0 = +0.17$)

b) The $r_\mathrm{E} - r_\mathrm{D}$ distribution

The parameter $r_\mathrm{E} - r_\mathrm{D}$ is simply the distance in AU that the Earth passes inside the nominal node of the dust trail (between the Sun and the trail, neg. $r_\mathrm{E} - r_\mathrm{D}$), or outside (pos. $r_\mathrm{E} - r_\mathrm{D}$).

As discussed in Section 2, the profile in $r_\mathrm{E} - r_\mathrm{D}$ is a function of Δa_0, age and ejection processes. Overall, it displays a skewed distribution towards increasing $r_\mathrm{E} - r_\mathrm{D}$. We have continued using a Gaussian fit to the profile in $r_\mathrm{E} - r_\mathrm{D}$, due to the lack of any quantified theoretical expression for how it varies with these dependent quantities. As trails age and diffuse, they are expected to become more Gaussian in profile, losing their sharp central core.

In making the fit to the data, $r_\mathrm{T} - r_\mathrm{D}$ is used in which r_E has been modified for the topocentric coordinates of the observer. In every case, this must make the value of $r_\mathrm{E} - r_\mathrm{D}$ become more positive, as visual observations are necessarily made from the night side of the Earth. The form of the correction is

$$r_\mathrm{T} - r_\mathrm{D} = (r_\mathrm{E} - r_\mathrm{D}) + 0.000043\ \sin(\text{solar depression angle})$$

where the solar depression is the angle the Sun is below the horizon and the constant is the radius of the Earth in AU. In 1866, the peak observed from the UK had a solar depression angle of 52° giving an additional +0.000034 AU to $r_\mathrm{E} - r_\mathrm{D}$. In all other years, the correction is smaller and for all predictions a value of +0.00002 AU is used.

c) ageing

Any process that results in diffusion of the dust trail, beyond the stretching of the trail, will lower the peak trail and widen the profile. On the assumption that this effect is uniform with time, the loss of the peak intensity should be $y^{\mathrm{age}-1}$, where age is given in revolutions, and y is the derived constant.

4. Dust Trail data

Calculation of dust trail parameters requires an accurate knowledge of the parent comet's orbit at the perihelia during which the dust trails were created. We used two orbits calculated by Nakano, one (55P-orb1) derived from modern observations and the other (55P-orb2) incorporating positions from the 1366 passage. Integrations were performed with the Mercury package. Table 1 gives the derived dust trail parameters using orbit 55P-orb1. The data for the 20-rev trail in 1998 is from 55P-orb2 which would be more reliable for the 1333 ejection.

5. Linear and non-linear dust trail encounters

A dust trail that evolves solely under the influence of solar gravity and srp will be uniform and pass from slightly inside and in front of the comet's orbit (shorter orbital periods), through the comet's position, to well behind and outside the comet's orbit (longer orbital periods). The bulk of the particles will be behind and outside the comet's orbit due to the effects of srp as mentioned above. Such a dust trail is linear in that the density, nodal position and cross-section are uniformly changing functions from one revolution to another and from year to year. Any disruption to the dust trail can result in erratic and unpredictable variations in sections of the trail and encounters at or near these sections are non-linear.

Due to the positioning of the dust trail nodes close to the Earth's orbit, the Earth can disrupt a small section of a dust trail once every year during the period the dust trail is passing through the node. While other planets can cause a more general bulk shift in the trail, the greatest cause of non-linearities is when the Earth passes through the center of the dust trail. This physically removes those particles that become meteors and scatters those that have very close approaches, but above the atmosphere. However at greater distances, the disruption is more orderly and can be calculated. The result is a complex region of stretching, compression, folding, tilting and scattering within a small section of the dust trail. Whilst a nominal dust trail center and density can be calculated for non-linear encounters, the effects described make it highly unreliable to assume these parameters apply to the region of the disrupted section actually encountered, at some specific $r_\mathrm{E} - r_\mathrm{D}$.

Table 1 – Dust trail parameters using orbit 55P-orb1 (from observations 1866–1998 by Nakano MPC 31070) r_{E} calculated using geocenter from DE403.

Year	Month	Day	Solar Long.	Revs	Δa_0	$r_{\mathrm{E}} - r_{\mathrm{D}}$	f_M
1833	11	13.435	233.184	1	0.174	−0.00021	1.028
1866	11	14.047	233.334	4	0.059	−0.00030	0.389
1867	11	14.394	233.421	1	0.373	−0.00015	1.043
1869	11	14.019	233.539	3	0.320	−0.00048	0.470
1966	11	17.497	235.159	2	0.168	−0.00013	0.535
1969	11	17.375	235.263	1	0.934	−0.00004	1.097
1998	11	17.013	234.460	20	−0.023	−0.00015	–
1999	11	18.091	235.292	3	0.138	−0.00067	0.398
2000	11	17.329	235.267	2	0.302	−0.00119	0.574
2000	11	18.149	236.096	8	0.066	0.00069	−0.138
2000	11	18.155	236.102	8	0.064	0.00077	0.292
2000	11	18.168	236.115	8	0.060	0.00086	−0.049
2000	11	18.328	236.276	4	0.114	0.00078	0.138
2001	11	18.387	236.083	7	0.092	−0.00082	−0.003
2001	11	18.418	236.114	7	0.081	−0.00044	0.157
2001	11	18.463	236.160	7	0.068	−0.00005	−0.005
2001	11	18.729	236.428	9	0.041	0.00015	0.395
2001	11	18.765	236.464	4	0.142	0.00023	0.139
2001	11	18.816	236.516	9	0.047	−0.00059	−0.017
2001	11	18.870	236.570	9	0.050	−0.00090	−0.005
2002	11	19.167	236.610	7	0.113	−0.00015	0.132
2002	11	19.180	236.623	7	0.093	0.00012	−0.004
2002	11	19.444	236.890	4	0.172	−0.00005	0.151

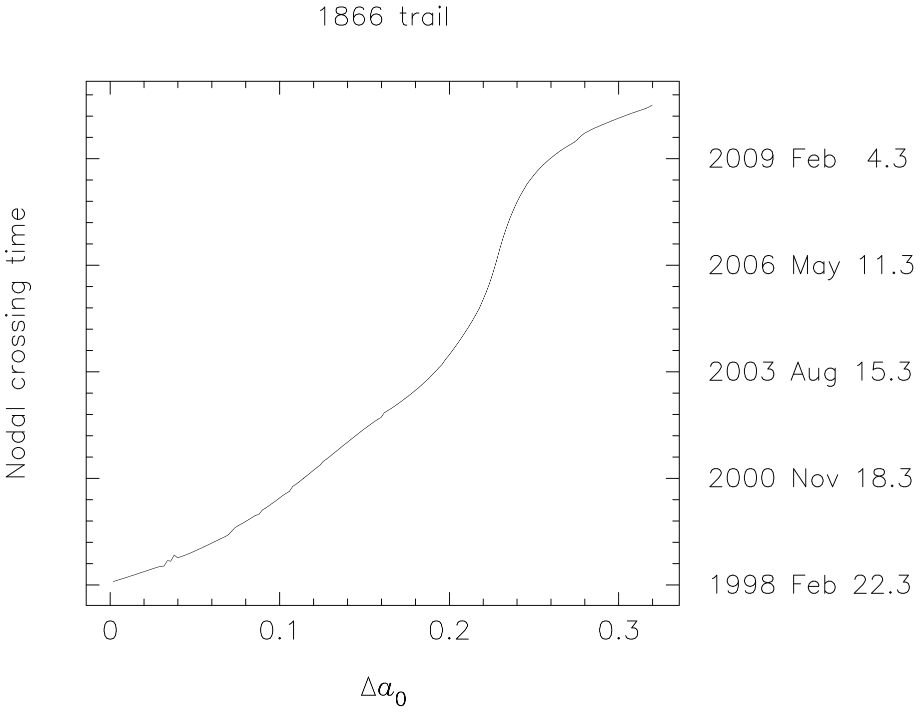

Figure 2 – Undisturbed 4-rev dust trail ejected in 1866.

Calculating the full nature of these effects over the whole dust trail section requires considerable effort and is equivalent to the sort of work done by software like JPL's SENTRY and University of Pisa's Clomon, in calculating impact probabilities of asteroids for non-linear encounters with the Earth.

The 4-rev dust trail (ejection in 1866) has had no close encounters with the Earth since it was formed. As can be seen in Figure 2, encounters at the present epoch will be linear. The situation is very different for the 7-rev trail (ejection in 1767) shown in Figure 3. Numerous disruptions are caused by the Earth's passage close to the trail. Those to the left of the plot resulted from the encounters listed in Table 2.

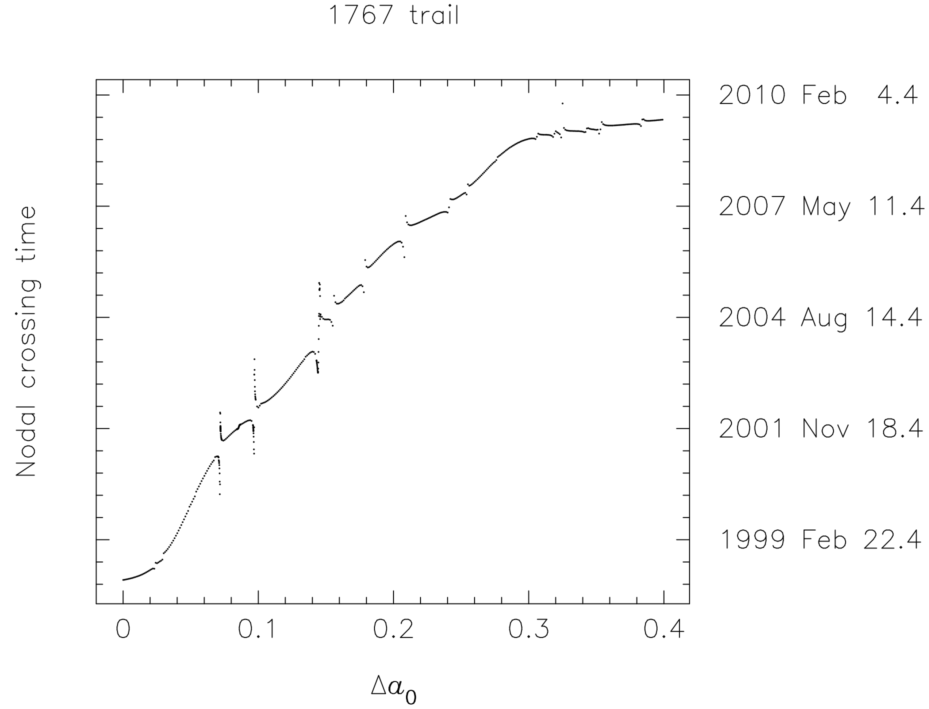

Figure 3 – Disruptions of the 7-rev dust trail ejected in 1767 due to close encounters with the Earth.

Table 2 – Parameters for some sections of the 7-rev (1767) trail that are disrupted by the Earth's passage in the years stated. Δa_0 is the difference in semi-major axis between the ejected particle and the comet at the time of ejection. $r_\mathrm{E} - r_\mathrm{D}$ is the miss distance by the Earth from the nominal trail center in the year given.

Year	Revolutions	Δa_0	$r_\mathrm{E} - r_\mathrm{D}$
1932	5	0.024	+0.0055
1899	4	0.029	+0.0120
1866	3	0.072	+0.0013
1833	2	0.097	−0.0015
1800	1	0.145	+0.0029
1867	3	0.155	−0.0056
1901	4	0.178	+0.0048
1834	2	0.208	−0.0042
1868	3	0.241	−0.0094
1835	2	0.319	−0.0064
1869	3	0.325	−0.0005
1801	1	0.353	+0.0030

The slope of the plotted line is inversely proportion to the trail density (f_M). Disrupted sections with near vertical lines have very low density with f_M approaching zero. A section that is horizontal would indicate a resonance with all orbital periods (represented by the range of Δa_0 in the horizontal section) coming back at the same time, although not necessarily at the same nodal distance. In the 1699 trail (Figure 4), there is a critical value of Δa_0 above which particles find themselves in the 3:1 resonance with Jupiter, hence that branch of the trail gets well seperated from the main part.

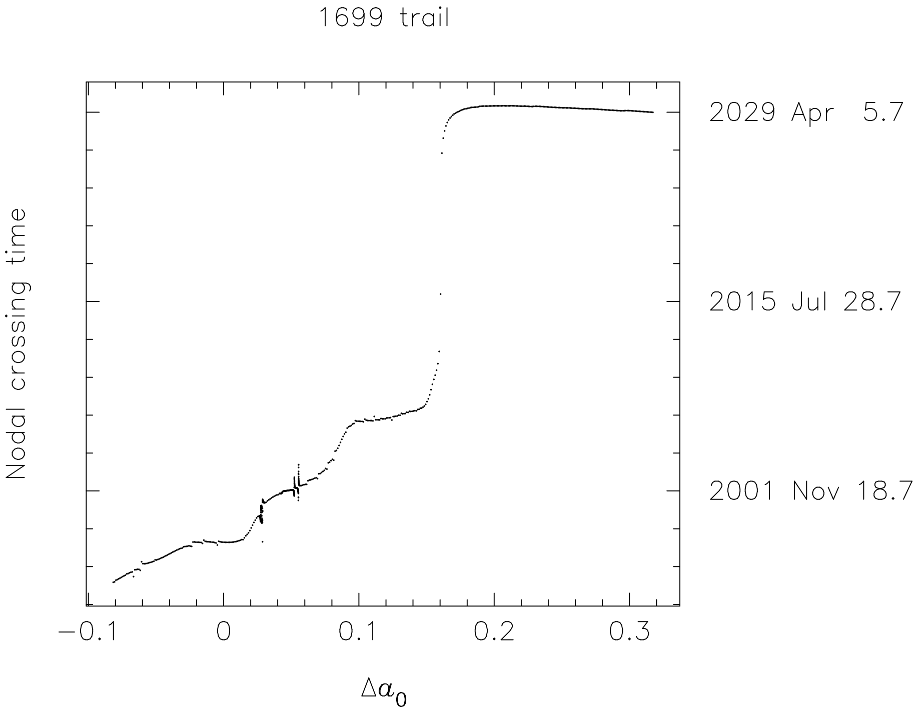

Figure 4 – Disruptions of the 9-rev dust trail ejected in 1699.

6. Prediction timing accuracy

Accurate methods of observation can derive the time of maximum of a meteor outburst to better than 3 minutes. This is better than the level of the prediction accuracy given that there is a difference of several minutes in calculating dust trail nodal longitudes using the two different orbits for 55P/Tempel-Tuttle. Use of these different orbits gives small differences in other dust trail parameters (Δa_0, $r_{\mathrm{E}} - r_{\mathrm{D}}$ and f_M) but these make little difference in the ZHR and outburst width calculations. It is useful to include the nodal longitudes derived using orbit 55P-orb2 to give two sets of residuals in the timing of maxima. In Table 3, the UT time and nodal longitude (C_2) are given from 55P-orb2 and residuals for both this and 55P-orb1 are $O - C_2$ and $O - C_1$ respectively. C_1 is given as the longitude in Table 1.

Two linear encounters, the 4-rev in 2000 and 2001, are possibly affected by the maxima of other trails nearby (the broad 8-rev in 2000 and the broad 9-rev in 2001). For the 2001 encounter, the same derived time of maximum is given by Arlt et al. [14] and by Uchiyama [15], the latter separating it from the 9-rev trail on the basis of differences in the population index. The 2001 4-rev peak time gives residuals less than 10 minutes. The analysis of the 2000 4-rev peak does not try to seperate the influence of the broad 8-rev trail and this may have pulled the 4-rev to an earlier peak (the $O - C$ is around 33 mins). Conversely, the 4-rev peak may have had less effect on the 8-rev timing; the 4-rev having a rather shorter duration. The small $O - C$ of the 8-rev is possibly fortuitous however, given that the encounter is non-linear and the peak poorly defined.

Table 3 – N = non-linear dust trail section, L = linear dust trail section. The observed (O, topocentric) longitude of the peak is taken from the reference in the last column. In cases where no topocentric correction to the time of maximum is made, or no mention is made of any such correction, the reported value has been corrected using the formula in [3] before inclusion in this table.
A "?" appears against values that are uncertain. This uncertainty takes two forms. Firstly, predictions may be uncertain due to the non-linearity of the dust trail and secondly, the maxima of several dust trails may be close together in time and influence the calculation of the time of the observed maxima.
The mean absolute error for eight encounters with no influencing factors (only one, weighted, $O-C$ derived for each encounter) is $|O-C_1| = 0.^{\circ}002$ and $|O-C_2| = 0.^{\circ}005$, being 3 minutes and 7 minutes respectively. The maximum error is 12 minutes for the 1999 storm using orbit 55P-orb1.

Node (Orbit C_2) in UT	Rev	O (topocentric)	C_2	$O-C_1$	$O-C_2$	Ref.
1833 11 13.429	1 L	233.15 ?	233.178	−0.022?	−0.028?	[5]
1866 11 14.041	4 L	233.334	233.328	0.000	+0.006	[5]
1867 11 14.389	1 L	233.423	233.416	+0.002	+0.007	[5]
1869 11 14.013	3 L	233.540	233.532	+0.001	+0.008	[1,6]
1966 11 17.493	2 L	235.159	235.155	0.000	+0.004	[5]
1969 11 17.370	1 L	235.276	235.268	+0.003	+0.008	[5]
		235.271		−0.002	+0.003	[7]
1999 11 18.086	3 L	235.285 ± 0.001	235.287	−0.007	−0.002	[8]
		235.284 ± 0.001		−0.008	−0.003	[9]
		235.283 ± 0.002		−0.009	−0.004	[10]
		235.285 ± 0.001		−0.007	−0.002	[11]
2000 11 17.324	2 L	235.28 ± 0.01	235.264	+0.011	+0.016	[12]
		235.266 ± 0.003		−0.003	+0.002	[13]
18.143	8 N		↗ 236.090			
		236.09 ± 0.01		−0.003?	+0.003?	[12]
18.149	8 N		↘ 236.096			
18.322	4 L	236.25 ± 0.01	236.270	−0.026?	−0.020?	[12]
2001 11 18.412	7 N	236.137 ± 0.003	236.108	+0.023?	+0.029?	[14]
		236.154 ± 0.003		+0.040?	+0.046?	[14]
18.724	9 N	236.431	236.423	+0.003?	+0.008?	[15]
18.758	4 L	236.458	236.457	−0.006?	+0.001?	[15]
		236.458 ± 0.003		−0.006?	+0.001?	[14]

The final two non-linear encounters are the 7 and 9-rev in 2001. The analysis by Uchiyama [15] has some uncertainty in the time of the 9-rev peak due to uncertainties in determining the appropriate population indices for the two trails, but his derived time of maximum is perhaps reliable to around ±7 minutes. The $O-C$s of 4 and 12 minutes against the two orbits is again pleasing but again possibly fortuitous. Other encounters with the 9, 10 and 11-rev trails in the hours after the main 9-rev encounter would act to skew the time of the 9-rev maximum and extend the declining branch. From the ZHR analysis below, these other trails would only contribute a total ZHR of up to 50 so would tend to extend the trailing branch of the derived 9-rev activity curve rather than shift the peak.

The observed activity from the 7-rev trail in 2001 was unusual. This non-linear encounter had a pronounced double peak and occured late by many tens of minutes. The suggestion [14] that the double peak represents passage through a hollow tube of material is untenable; a coherent tube being dynamically impossible. Even in our initial 1999 paper, it was clear that this encounter would be less reliable than others and it seems certain that detailed dynamical calculations are required before any useful conclusions about this encounter can be made. The main 7-rev encounter in 2001 appears in Figure 3 as a kink in the middle of on otherwise linear section of the dust trail (refer also to Table 2.).

The large residual for the 1833 storm is discussed in the ZHR Section below.

The times of the storm peaks in 2002 given in the abstract are a mean of predictions from both orbits.

7. ZHR fit

Most of the necessary discussion regarding the model fit of observed ZHR to calculated dust trail density is discussed in Section 3. Despite the caveats contained in that section, it is reasonable to fit a double Gaussian to the Δa_0 vs. $r_{\mathrm{E}} - r_{\mathrm{D}}$ parameter space. Data from twelve dust trail encounters are used in the fit. The input data and calculated ZHRs are given in Table 4.

Table 4 – ZHR and Width data and predictions using the dust trail parameters in Table 1. References for the data in specific years are the same as in Table 2. Weight has been arbitrarily assigned according to the quality of the observations, presence of moonlight, non-linearity of the observed trail or interference from other trails. (Nine reliable shower widths are used in the width fit, all with equal weight, the 2001 7-rev being excluded.) The 2001 4 and 9-rev observed ZHRs are derived from Uchiyama [15] and scaled (by a factor of 1.1) to give the *IMO* ZHR at maximum [14]. Calculations repeated using 55P-orb2 resulted in changes in derived ZHRs and widths of only a few percent.

Year	Rev	Obs. ZHR	Weight	Calc. ZHR	Width	
					Obs. FWHM	Calc. FWHM
1801	2 L			4200		52
1833	1 L	(60000)	0	68000		42
1866	4 L	8000	2	7900	59	55
1867	1 L	4500	2	4700	75	55
1869	3 L	(1000)	1	1200		74
1966	2 L	90000	4	25000	38	50
1969	1 L				68	70
1998	20 N				720	(780)
1999	3 L	3700	8	4000	50	56
2000	2 L	100	4	87	71	61
2000	8a N	135	2	120	(210)	(110)
2000	8b N	135	2	170	(210)	(110)
2000	8c N			18		(110)
2000	4 L	450	4	460	(160)	63
2001	7a N			3		(98)
2001	7b N	1620	4	600	85	(95)
2001	7c N			33		(92)
2001	9a N	960	2	920	225	(120)
2001	11			18		170
2001	4 L	2450	8	2600	68	66
2001	10			18		140
2001	9b N			16		(120)
2001	9c N			1		(120)
2002	7a L			810–2000		105
2002	7b N			25–60		(100)
2002	4 L			2900–6000		71

It is understandable that the 1833 observed ZHR derived by Brown [5] is unreliable, all observations being fortuitous and, at the time, of a largely unknown phenomenon. The peak ZHR is probably a gross underestimation due to the calculated peak occuring some 45 mins after the derived "observed" maximum which occured at morning twilight for the observers in eastern N. America. A significantly increased ZHR for 1833 (possibly of the order of 150 000+) and the fact that the fit here calculates too low a ZHR for 1966 (by a factor of three) suggests that the 4-rev trail in 2002 is probably underpredicted. An enhanced peak in the model parameter space would occur at a certain Δa_0 (modified by β) for young dust trails when production of typical visual sized particles peak at ejection velocities which favour this Δa_0. The near-central 1867 1-rev encounter, however, appears to fit the model well. This observed peak ZHR in 1867 may not be reliable and this encounter is at a much larger Δa_0 which may be a significant difference. Alternatively 1867 represents a problem for the above speculation.

The use of a Gaussian in $r_\mathrm{E} - r_\mathrm{D}$ gives an adequate fit to the input data other than 1966 (mentioned above) and 2001 7-rev. As the 2001 non-linear 7-rev trail encounter had the maximum time significantly different from prediction, this implies that the use of its nominal dust trail parameters is unwarranted. As a result, the poor ZHR fit for this trail need not be a problem. This is discussed more below.

8. ZHR predictions for 2002

2002 4-rev

This trail possibly occurs closer to the peak in $r_\mathrm{E} - r_\mathrm{D}$ than any previous dust trail encounter in the last 200 years. Being very close to the 1833 and 1966 encounters (in parameter space) it is reasonable to expect this encounter to be underpredicted in the same way as these young trails, although of lesser magnitude due to the greater age. Whereas a factor of three seems applicable for the excess ZHR of 1833 and 1966, a factor of two might be more reasonable for this 4-rev trail. Other encounters with this dust trail in 2000 and 2001 suggest this dust trail has behaved as a "normal" trail in those encounters and thus gives no reason to scale the ZHR for unusual cometary activity at formation.

2002 7-rev

Due to the non-linearity of the encounter in 2001, it is difficult to infer the 2002 7-rev activity based on last year's encounter. With an observed ZHR of 1620 and the best fit giving 600, this is too low by a factor of ~ 2.5. Assuming the fit does not significantly depart from the double Gaussian in this region of parameter space (as evidenced by the good fits for 1866 and 1999), there could be two reasons for this poor result:

a)—the non-linearity acts to increase the ZHR in the 2001 encounter. Detailed calculations are necessary to interpret what the overall dust-trail shape and density would be for a non-linear encounter. We know that the density (from f_M) had a sharply increasing value at the node, and diffusion of the particles would act to increase the density at the node (as diffusion in any "gas" would cause an increased density in regions adjacent to higher density, until an equilibrium is reached). Also, the dust trail parameters, calculated for the nominal center, would be unreliable for this trail. More so, the parameters of the trail at the encountered $r_\mathrm{E} - r_\mathrm{D}$ must be significantly less reliable.

b)—the trail ejected in 1767 may be denser than the average trail. If so, the ZHR for other 7-rev encounters should perhaps be increased by a factor of ~ 2.5. This same trail produced the 1869 storm, but there is no evidence from the existing, albeit poor quality, observations that the shower was overstrength in that year. However, without any theoretical resolution to this situation it must be considered possible that the 2002 encounter will be stronger in proportion to that of last year.

Like the 4-rev trail, this is also a near central encounter but being of greater age and with a different Δa_0, there seems no need to apply any significant adjustment for an enhanced core density.

Despite the probably higher ZHR of the 4-rev trail in 2002, the lower Δa_0 of the 7 rev trail (+0.11 as opposed to +0.17) will result in a higher proportion of bright meteors. This will have a marked bearing on observed meteors for lower limiting magnitudes as would be expected in full moonlight.

9. Outburst widths

Observed widths appear in Table 4. A fit using the same parameters as the ZHR model was reasonably successful. It took the form

$$\mathrm{FWHM} = x_1^{\mathrm{age}-1}(x_2 + x_3|\Delta a_0| + x_4 \Delta a_0^2)$$

but this was a more or less arbitrary choise of function. The basis for the form chosen was as follows:

Higher values of Δa_0 (both positive and negative values) show a correlation with higher ejection velocities although there would be some magnitude dependence due to the influence of srp. This effect was modelled by linear and quadratic terms in Δa_0 although in the fit, the quadratic term is insignificant.

Width appeared to show no dependence on $r_\mathrm{E} - r_\mathrm{D}$ after trialing various relationships. All showed a flat profile in this term without improving the residuals.

Diffusion of the dust trail with age clearly occurs as evidenced by the improved ZHR fit when such a parameter is included. The fit gave $x_1 = 1.19$ implying a 19% increase in width from return to return. Inclusion of the resonant section of the 20-rev trail in 1998 was included to show that in principal such trails can be considered without dramatically affecting the fit. Fits that excluded the 1998 data showed no overall improvement and some small anomalies existed. With the 1998 data included there appears to be some divergence from the fit for the 8-rev trail in 2000 and the 9-rev in 2002, the FWHM being under-predicted. Both these trail sections are non-linear, the observed FWHM is affected by the higher peak of the 4-rev trail in both years and the 9-rev prossibly has the declining branch overestimated due to the influence of several older trail that peaked after the main 9-rev trail. Despite these possible influences, it can be reasonably argued that the 7-rev trail will be broader than the the 105 mins given by the fit and may be closer to a FWHM of 150 min. A value of 130 min would seem a reasonable working figure.

For 2002, the 4-rev trail seems well predicted with a FWHM of 71 min.

10. Conclusion

The standard dust trail theory using rigorous dynamical calculations, including srp, give a good fit to the time of maximum and a reasonable fit to the ZHR and width of Leonid outbursts and storms. The ZHR peak in the parameter space of our density model appears to be too shallow when dealing with central encounters of young dust trails; a conclusion that has some theoretical basis. Non-linear encounters require considerably more detailed dynamical computations which were beyond the scope of this work. Conclusions based on the nominal dust trail parameters for non-linear encounters must be considered unreliable.

The two major encounters in 2002 are both linear and should be well predicted to within about 10 minutes and of the order of ±5 minutes using the derived dust trail parameters. Some uncertainty in the peak ZHR exists for both these trails that could increase the predictions by up to a factor of three. For the 7-rev trail over European longitudes the uncertainty results from the high ZHR from the same trail in 2001. Overall, it does not appear warranted to assume the observed activity of the non-linear encounter in 2001 should automatically imply higher than nominal rates in 2002, but without very extensive calculations we cannot deny this possibility. The 4-rev trail over N. American longitudes falls in roughly the same ZHR parameter space as the 1833 and 1966 Leonid storms. Given that both these storms seem rather underpredicted by our ZHR model, and bearing in mind that these are the only two linear encounters that are so badly predicted, it seems reasonable that the 4-rev encounter in 2002 could be double the nominal ZHR prediction.

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