Meteor outburst profiles and cometary ejection models

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Abstract. The spatial structure of meteor streams, and the activity profiles of their corresponding meteor showers, depend firstly on the distribution of meteoroid orbits soon after ejection from the parent comet nucleus, and secondly on the subsequent dynamical evolution. The latter increases in importance as more time elapses. For younger structures within streams, notably the dust trails that cause sharp meteor outbursts, it is the cometary ejection model (meteoroid production rate as a function of time through the several months of the comet's perihelion return, and velocity distribution of the meteoroids released) that primarily determines the shape and width of the trail structure. This paper describes how a trail cross section can be calculated once an ejection model has been assumed. Such calculations, if made for a range of ejection model parameters and compared with observed parameters of storms and outbursts, can be used to constrain quantitatively the process of meteoroid ejection from the nucleus, including the mass distribution of ejected meteoroids.

Keywords: Celestial mechanics · Comets · Dust trails · Leonids · Meteor outbursts · Meteor streams

1 Introduction: dust trail theories

Dust trail theories have been highly successful in predicting the sharpest storms and outbursts in the Leonids (Kondrat'eva & Reznikov 1985; Kondrat'eva et al. 1997; Asher 1999; Lyytinen 1999; McNaught & Asher 1999) and other streams (e.g. Reznikov 1983, 1993; Watanabe et al. 2005). During a single perihelion return of an active comet, meteoroids are released on to a range of orbits close to, but not identical to, the comet's orbit. The range of orbital periods soon causes the particles to stretch into a trail, which can already be quite long after just a few revolutions.

Even within one revolution, the orbits are subject to gravitational perturbations. The key realisation in the development of dust trail theories is that the perturbations are a function only of location along the trail, over short timescales (but at least for a few revolutions). This means that the determination of when outbursts occur reduces to a problem with two end points: the time t_0 when particles are ejected from the comet, and a later time t_1 when the Earth passes through the stream. The trail can produce an outburst at time t_1 if the result of planetary perturbations has been to bring the node of the trail particle orbits precisely to Earth intersection, rather than the node being inside or outside the Earth's orbit. Only one value of the orbital period allows particles to reach their node at time t_1. The idea of dust trail theories is to quickly find (iteratively) this value of the period, and then to calculate the perturbations on just one representative particle with that period. All particles with the same period have comoved between t_0 and t_1, i.e. have continuously been at almost the same point in space as each other, and have therefore been subject to the same perturbing accelerations from the planets.

A set of comoving particles are by definition at a single point along a trail. Although their orbital periods must be the same, their other orbital elements can differ owing to their range of ejection velocities. This leads to the trail having a nonzero width, generally much less than a trail's length but significantly greater than the size of the Earth. A trail is also much narrower than the width of the whole stream derived from the parent comet, as the stream has formed from meteoroids ejected over a long timescale, during which their orbits can diverge to a greater extent.

Over short enough timescales, however, the perturbations on a set of comoving particles are the same and trail cross sections are invariant. The width at a single point along a trail results from ejection velocities, not from differential perturbations. This allows calculations as described below. Such calculations are only applicable to young enough trails. In other cases, more detailed modelling of the stream is necessary (cf. Vaubaillon et al. 2005a,b). For the Leonids, trails whose age is a few revolutions (up to $\sim$10) are young enough (see Sec. 3). For other streams, the limiting age depends on the effect of perturbations, and thus especially on factors such as the proximity of the stream orbit to Jupiter. For long period streams (Lyytinen & Jenniskens 2003), a further consideration is the extreme sensitivity of the semi-major axis to perturbing forces; with showers such as the α-Aurigids (Jenniskens & Vaubaillon 2007) this can be relevant.

Some authors have used the parameter Δa_0 to indicate distance along a trail. This is the difference in semi-major axis (i.e. equivalent to period) from the comet at time t_0. Owing to planetary perturbations, the semi-major axis is not constant, even over a few revolutions. As radiation pressure (Sec. 2) affects the semi-major axis, it is easiest to define Δa_0 as being for particles on which radiation pressure is negligible, and to think of all comoving particles as having the same Δa_0, even though their semi-major axis can differ if they are subject to radiation pressure.

2 Method: calculating cross sections

In principle, observations of meteor showers can be used to place strong constraints on where along the parent comet's orbit meteoroids were released, or on the meteoroids' ejection velocities (e.g. Brown & Arlt 2000; Ma & Williams 2001; Ryabova 2001, 2007; Arter & Williams 2002; Asher & Emel'yanenko 2002). The available detailed observations of many storms and outbursts provide excellent opportunities in this regard. The requirement of the modelling is to reliably convert circumstances of ejection into observable quantities such as meteor activity profiles.

Provided that ejection speeds are small relative to the orbital speed (true in practice), calculations of a family of orbits of ejected particles can be linearised. From Lagrange's planetary equations, expressions for changes in orbital elements can be derived as functions of three ejection velocity components $(\mathit{\Delta} v_R, \mathit{\Delta} v_T, \mathit{\Delta} v_N)$, in radial, transverse and normal directions:

$$\begin{aligned} \mathit{\Delta} a &= \mathtt{AR}\,\mathit{\Delta} v_R + \mathtt{AT}\,\mathit{\Delta} v_T \\ \mathit{\Delta} r_D &= \mathtt{RR}\,\mathit{\Delta} v_R + \mathtt{RT}\,\mathit{\Delta} v_T + \mathtt{RN}\,\mathit{\Delta} v_N \\ \mathit{\Delta}\mathit{\Omega} &= \mathtt{ON}\,\mathit{\Delta} v_N \end{aligned} \tag{1}$$

where a = semi-major axis, r_D = heliocentric distance of descending node (the relevant node for Leonids) and $\mathit{\Omega}$ = longitude of ascending node. The coefficients `AR` etc. are functions of the elements (including the true anomaly ν) given in celestial mechanics books (e.g. Murray & Dermott 1999 Chap. 2; Danby 1988 Chap. 11; Roy 1988 Chap. 6; Bate et al. 1971 Chap. 9). As r_D is not one of the standard elements, $\mathit{\Delta} r_D$ can be derived from expressions for the changes in eccentricity e and argument of perihelion ω:

$$r_D = \frac{a(1-e^2)}{1 - e\cos\omega}$$

$$\mathit{\Delta} r_D = \frac{\partial r_D}{\partial a}\mathit{\Delta} a + \frac{\partial r_D}{\partial e}\mathit{\Delta} e + \frac{\partial r_D}{\partial \omega}\mathit{\Delta}\omega$$

(see also Pecina & Šimek 1997).

An outburst profile is determined by the ejection velocity distribution and the meteoroid production rate as a function of ν (Ryabova 2001). Any specific Earth encounter with a trail is parametrised by a single value of $\mathit{\Delta} a_0$ (Sec. 1). The Earth's passage through the trail occurs at a single value of $\mathit{\Delta} r_D$. It therefore suffices to calculate results along just one dimension, i.e. $\mathit{\Delta}\mathit{\Omega}$, which varies as the Earth moves through space and meteors are recorded, although we may calculate results in the $(\mathit{\Delta} r_D, \mathit{\Delta}\mathit{\Omega})$ plane if we wish to develop a picture of trail cross sections. The parameter $\mathit{\Delta} a_0$ can be thought of as measuring the along trail dimension, with $\mathit{\Delta} r_D$ and $\mathit{\Delta}\mathit{\Omega}$ spanning the across trail dimension.

In meteor outburst calculations, solar radiation pressure, parametrised by β, the ratio of radiation pressure to solar gravity, is less important than planetary perturbations. However, for the majority of particles that produce visual meteors, it is still significant. If $\beta = 0$, the value Δa in (1) equals Δa_0. When $\beta \neq 0$, the effect on the orbital period (critical for determining which particles comove and are therefore part of the same cross section along a given trail) can be calculated (Kondrat'eva & Reznikov 1985; Williams 1997; Asher & Emel'yanenko 2002), and so the correct value of Δa (i.e. that corresponds to the desired Δa_0) can be found, to be used in (1). This correct Δa is a function of ν. In addition, the coefficients `AR` etc. in (1) can be evaluated using the appropriate value of the central mass, for any given β.

For any ν the equations (1) are linear and therefore easily inverted to find $(\Delta v_R, \Delta v_T, \Delta v_N)$ for a given $(\Delta a, \Delta r_D, \Delta \Omega)$. An assumed distribution of particles in $(\Delta v_R, \Delta v_T, \Delta v_N)$ space therefore yields a density of particles, ejected at that ν, that reach the point $(\Delta a, \Delta r_D, \Delta \Omega)$ in the trail. An assumed meteoroid production rate then allows the contributions from different ν to be summed, and the overall density for the given ejection model at the given $(\Delta a_0, \Delta r_D, \Delta \Omega)$ to be evaluated. When integrating over ν, the Jacobian of the transformation from $(\Delta v_R, \Delta v_T, \Delta v_N)$ to $(\Delta a, \Delta r_D, \Delta \Omega)$ phase space is used for normalisation between different ν values. The Jacobian is a function of ν but not of $(\Delta v_R, \Delta v_T, \Delta v_N)$.

3 Application: Leonids

The above procedure allows density profiles to be quickly generated for a large number of models each with its own ejection parameters. The physics of the cometary mass loss and meteoroid ejection process (Whipple 1951; Jones 1995; Ma et al. 2002) determines the range of models that it is reasonable to consider. Observational data have been obtained for various Leonid storms and outbursts (e.g. Jenniskens et al. 2000). Work will soon be in progress to generate density profiles (for a range of models) relating to Leonid trail encounters for which observational results are available, and to make a careful comparison with observed data. Such an approach allows ejection parameters to be constrained (e.g. Brown & Jones 1998). In the case of the Leonids, it is particularly useful that many outbursts have been observed, as each ejection model can simultaneously yield density profiles for every outburst. This assumes the process of meteoroid production to be the same on each return of the parent comet that has given rise to a trail later encountered by the Earth, indeed the modelling process can be a test of whether this is true.

The procedure is computationally light (thus enabling a wide range of ejection models to be assessed), firstly because it consists of calculations only for those parts of trails encountered by the Earth, and secondly because it does not involve numerical integrations of orbit evolution. Integrations are required only (i) to determine the location in the ecliptic, relative to the Earth's orbit, of a single reference point in a trail cross section, sometimes referred to as the "trail centre" although cross sections are not symmetrical about the trail centre; (ii) to verify that trails being considered have cross sections

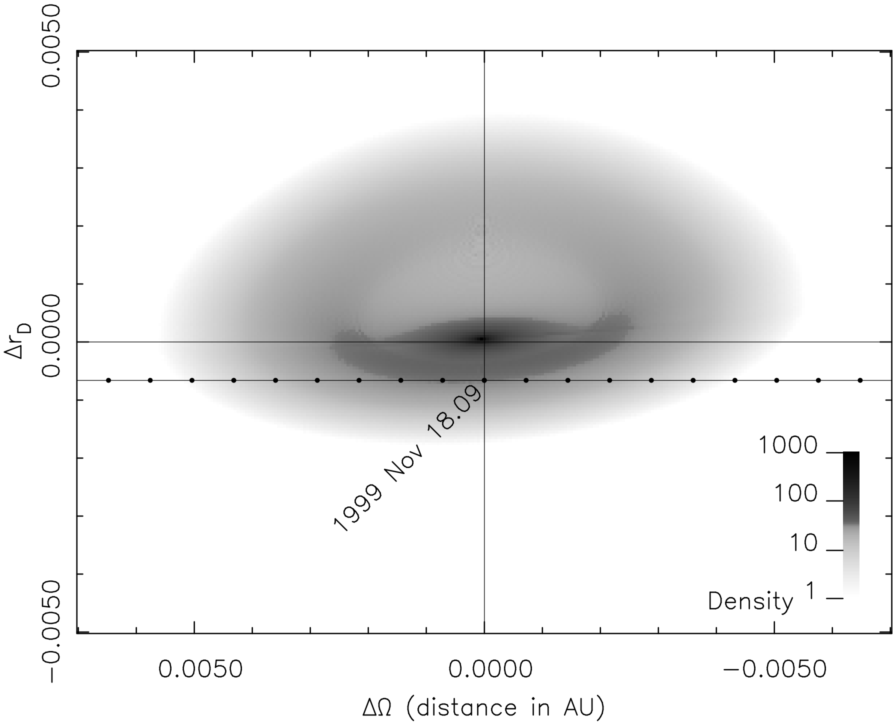

Fig. 1. Density in the ecliptic of the cross section of the 1899 Leonid trail encountered by the Earth in 1999, for one example ejection model. $\Delta\Omega$ converted from radians to distance in Astronomical Units. Radiation pressure parameter β=0.001. Meteoroid production rate uniform in true anomaly (cf. Kresák 1976; Brown & Jones 1998). Mean ejection speed $50/r$ m/s (r in AU), a power law with exponent -1 in heliocentric distance r being assumed for simplicity in this example, with a range around this mean value being allowed at any r. Ejection directions uniform over sunward hemisphere. Earth went from right to left, Earth shown actual size at 1 hour intervals.

that are invariant under the planetary perturbations that occur between the ejection epoch and the observed epoch; and (iii) to verify that perturbations occurring during the ejection epoch itself, which may last as long as a year or so, are negligible. Regarding (ii), Leonid trail cross sections seem to remain invariant for a few revolutions, although certainly not for as long as e.g. 20 revolutions, in general (cf. McNaught & Asher 1999; Asher 2005). Regarding (iii), some new test integrations using Everhart's (1985) RADAU integrator as implemented in Chambers' (1999) MERCURY package show that, as a result of perturbations during the ejection arc (taken as $r < 3.4$) alone, the nodal position of Leonid particles is rarely displaced by more than an Earth diameter or so, i.e. a very small distance compared to the entire trail cross section. Exact

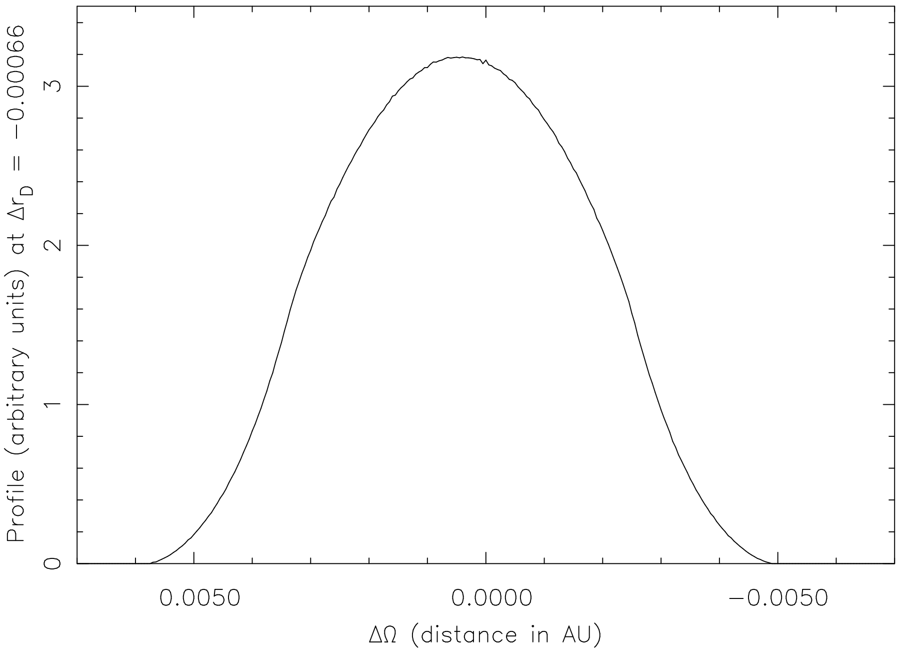

Fig. 2. The density profile encountered by the Earth as it goes through the trail cross section shown in Fig. 1.

displacements depend on the relative configuration of 55P/Tempel-Tuttle and Jupiter in their orbits on the given return of 55P.

An example cross section is shown in Fig. 1. For a single value of ν and a single ejection speed, the locus of points in the $(\mathit{\Delta} r_D, \mathit{\Delta\Omega})$ plane is an ellipse (cf. Kondrat'eva & Reznikov 1985; Müller et al. 2001; Welch 2003). The density distribution in Fig. 1 is essentially a sum of (non-concentric) ellipses, ejection occurring over an orbital arc spanning several months before and after perihelion, and with a range of ejection speeds at each ν. Future modelling will consider the ejection process in more detail, addressing for example the possibility that meteoroid production from 55P/Tempel-Tuttle concentrates strongly near perihelion, as suggested by observations of the coma (Watanabe et al. 2001).

When the Earth passes through the cross section shown in Fig. 1, it encounters a one dimensional profile as plotted in Fig. 2, i.e. the calculated data in Fig. 2 are basically a subset (for a single $\mathit{\Delta} r_D$ value) of those in Fig. 1. The width of the profile shown in Fig. 2 is several hours, clearly inconsistent with the accurately determined observed profile (Arlt et al. 1999), immediately showing that the ejection model adopted here for illustrative purposes does not match the real ejection process. A range of ejection models will soon be tested, and indeed as observational Leonid data exist for a different magnitude intervals, it should be possible to constrain the meteoroid production as a function of β (equivalently meteoroid mass) as well as ν.

At present it is possible to envisage either of two conclusions. On the one hand, this kind of modelling could provide good fits to sufficiently young (e.g. 1-revolution, or 2-revolution, or more) Leonid trails and thus give strong quantitative constraints on ejection processes. Alternatively this modelling could demonstrate that no ejection model can fit all the observations, suggesting that ejection processes alone do not fully determine trail cross sections even on short timescales, and thus verifying that other processes such as radiative forces during the orbital evolution are important even over short times (cf. Lyytinen & van Flandern 2000; Lyytinen et al. 2001). Either of these conclusions would be valuable in the study of dust trails and meteor outbursts.

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