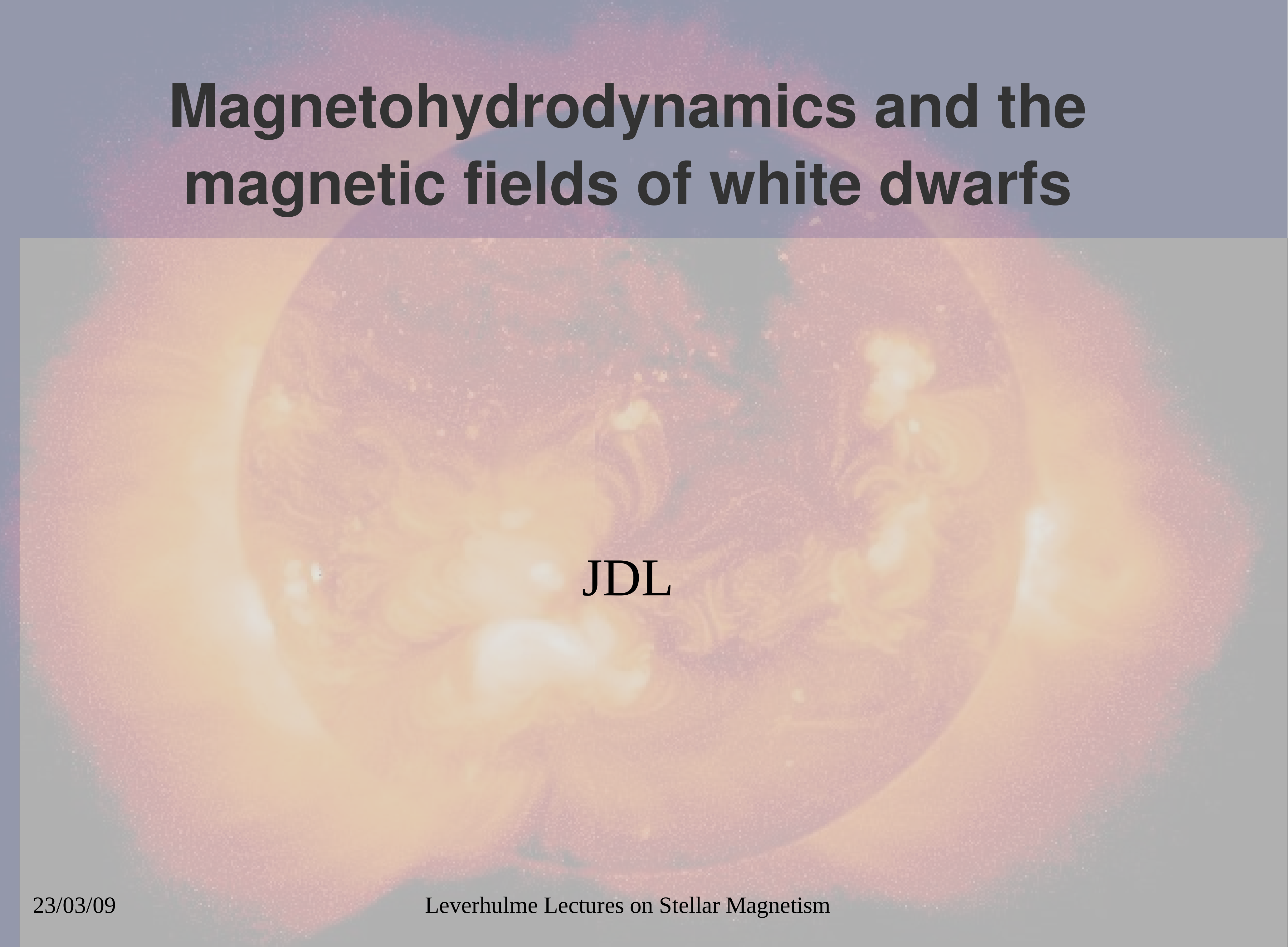
Magnetohydrodynamics and the magnetic fields of white dwarfs



JDL

Decay of large-scale magnetic fields

- We have seen that some upper main sequence stars host magnetic fields of global scale and dipolar topology
- These fields are found in Aps that are members of both young (~10^7 yr) and old (~10^9 yr) open clusters, so fields persist. Why?
- Maxwell's curl equations in limit of slow changes:

$$\nabla\times\boldsymbol{E}=-\left(\frac{1}{c}\right)\frac{\partial\boldsymbol{B}}{\partial t}\;\Rightarrow\;\frac{E}{L}\sim\frac{B}{c\tau}\qquad\nabla\times\boldsymbol{B}=\frac{4\pi\boldsymbol{j}}{c}\;\Rightarrow\;\frac{B}{L}\sim\frac{4\pi j}{c}$$

where L and τ are the characteristic size of the system and the timescale for change. Using Ohm's Law, $j = \sigma E$, and combining, we find $\tau \sim 4\pi\sigma L^2/c^2$.

- With $\sigma \sim 5\times10^7 T^{3/2}$ (cgs units), the timescale for decay of a global field of scale ~10^{11} cm is about 10^{18} s or 3×10^{10} s
- Could such slowly decaying fields persist after main sequence?

The induction equation, flux freezing, and field decay

- The field decay estimate assumes a static gas. We can get a much more general equation by combining Maxwell's equations

$$\nabla\times\boldsymbol{E}=-\left(\frac{1}{c}\right)\frac{\partial\boldsymbol{B}}{\partial t} \qquad \nabla\times\boldsymbol{B}=\frac{4\pi\boldsymbol{j}}{c}$$

with the general form of Ohm's law, $\boldsymbol{j}=\sigma[\boldsymbol{E}+(\boldsymbol{v}/c)\times\boldsymbol{B}]$

- Take the curl of the $\nabla\times\boldsymbol{B}$ equation using Ohm's law, notice that $\nabla\times\nabla\times\boldsymbol{B}=\nabla\cdot(\nabla\cdot\boldsymbol{B})-\nabla^2\boldsymbol{B}=-\nabla^2\boldsymbol{B}$, replace $\nabla\times\boldsymbol{E}$ from the other Maxwell equation, and finally get the *induction (MHD) equation*

$$\frac{\partial\boldsymbol{B}}{\partial t}=\nabla\times(\boldsymbol{v}\times\boldsymbol{B})+\left(\frac{c^2}{4\pi\sigma}\right)\nabla^2\boldsymbol{B}$$

- This equation gives us the same time scale for field decay as before if $\boldsymbol{v}=0$

Flux freezing

- From the induction equation we can show that flux freezing occurs
- Draw a closed curve around a magnetic flux *Φ* and let it move with local fluid velocity *v*. Then

$$\frac{d\phi}{dt}=\int_S \frac{\partial \boldsymbol{B}}{\partial t}\cdot \boldsymbol{n}\,dA+\oint \boldsymbol{B}\cdot(\boldsymbol{v}\times d\boldsymbol{l})=\int_S \left[\frac{\partial \boldsymbol{B}}{\partial t}-\nabla\times(\boldsymbol{v}\times\boldsymbol{B})\right]\cdot\boldsymbol{n}\,dA$$

- Now using induction equation on square brackets we find

$$\frac{d\phi}{dt}=\int_S\left(\frac{c^2}{4\pi\sigma}\right)\nabla^2\boldsymbol{B}\cdot\boldsymbol{n}\,dA$$

- Then flux through the closed curve changes only by the Ohmic diffusion term, and is practically constant when σ --> OO: the flux is *frozen* into the conducting plasma in which it is embedded
- This is a quick proof of an important general property

Physics of compact stars

- Before we looks at fields in compact stars, let's review their physics
- What provides support against gravity in MS or giant star?
- Why does a MS or giant star evolve?
- What happens to MS or giant star when nuclear fuel is exhausted?
- What are available final states of a star after fuel exhaustion? How do they depend on initial mass of evolving star?
- What provides pressure support in WD? How does effect vary with mass and internal temperature?
- Could you derive to order-of-magnitude that $P \sim \rho^{5/3} (h^2 / m_e m_A^{5/3})$ in a white dwarf?
- Does a white dwarf evolve? Why? How?
- Why is there a maximum mass for a WD?
- What is pressure support in a neutron star? How is it different from that of a white dwarf?

Evolution to white dwarf

- As main sequence star evolves to giant, develops deep convective envelope. Effect on field not clear, but some fields now detected in red giant stars
- Suppose that most magnetic flux lines are anchored in the stellar core. Evolution to final white dwarf state will reduce radius from R_{MS} to R_{WD} where the white dwarf has about 1% the radius of the MS star. The (roughly conserved) lines of flux will then be compressed into a much smaller equatorial surface area, and we estimate that B would vary approximately as $B_{WD}/B_{MS} \sim (R_{WD}/R_{MS})^{-2}$
- Hence a field of 10^3 G on MS might become $\sim 10^7$ G in a WD
- So we search for fields of this general size in WDs
- The same reasoning suggests that neutron stars, of $R \sim 10$ km $\sim 10^{-5} R_0$ could have fields of 10^{13} G

Field measurement for huge fields

- So we turn to methods of measuring magnetic fields in situations where they are really large (up to say 10^9 G, 1 GG, 100 kT)
- Fields of white dwarfs are observed using several distinct detection methods which correspond to the behaviour of atoms in increasingly large fields
    - For fields below about 100 kG, the normal Zeeman effect (and perhaps the Paschen-Back effect in H) are used as in non-degenerate stars
    - From 100 kG to about 10 MG, the linear Zeeman effect is overtaken by the quadratic Zeeman effect
    - Above 10 MG, even the spectrum of H is no longer easily recognised. It is greatly distorted, and continuum polarisation (circular and then linear) becomes detectable

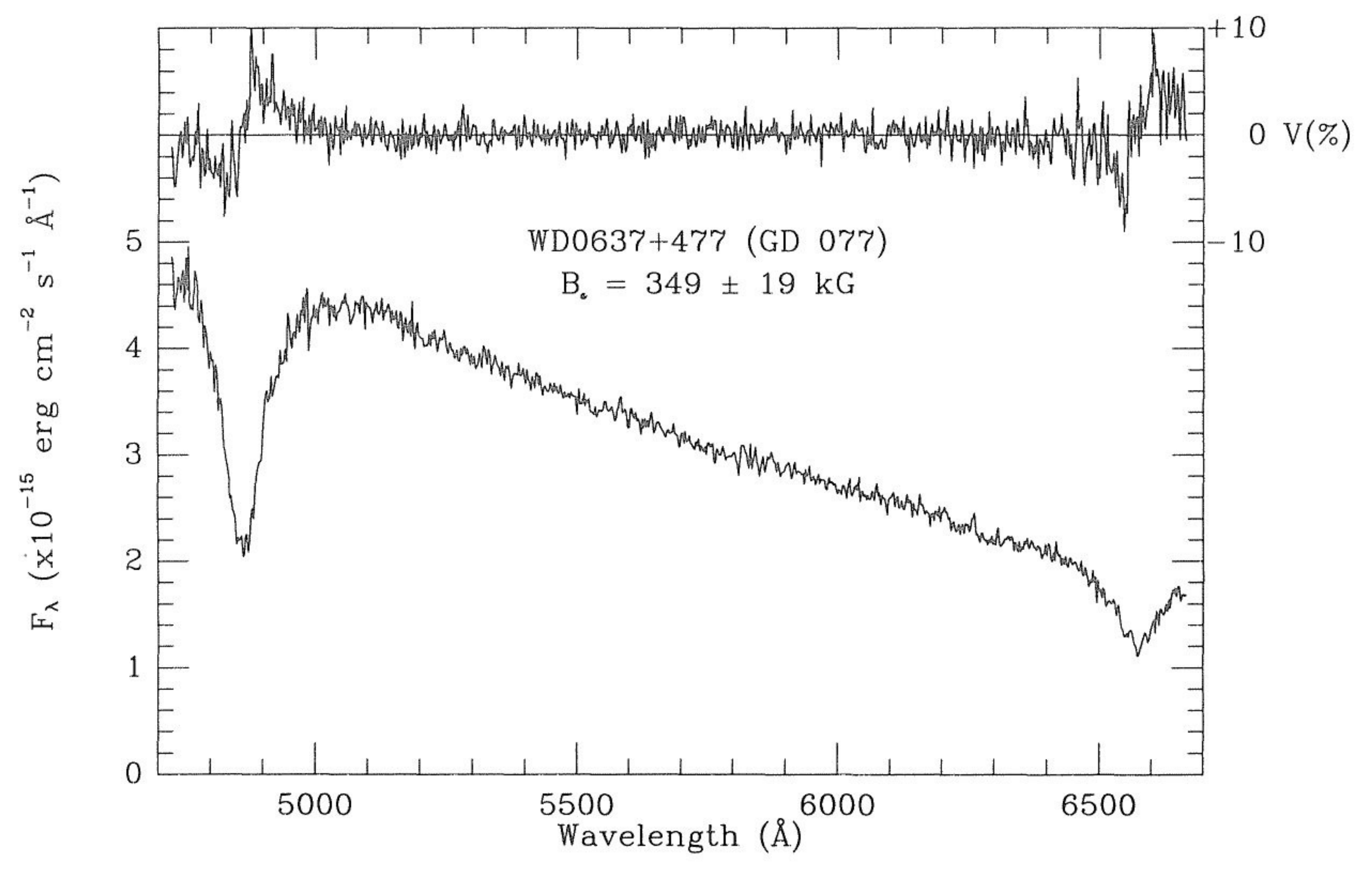
Quadratic Zeeman effect

- Recall that the Hamiltonian of an atom in a magnetic field has a linear and a quadratic term in B.
- The effect of the quadratic term is to shift all spectral line components in H to shorter wavelengths by about

$$\Delta\lambda_Q \simeq \left(-e^2 a_0^2/8mc^3h\right)\lambda^2 n^4\left(1+m_L^2\right)B^2$$

  where wavelengths are in Å, a_0 is the Bohr radius, and n and m_L are the principal and magnetic quantum numbers of the upper level
- The quadratic effect dominates for hydrogen H10 for B > 10 kG
- At 1 MG, H8 would be shifted by about 350 km/s relative to Hα, an easily detectable effect (Preston 1970, ApJ 160, L143)
- Polarisation effects are similar to those of Zeeman effect, but components do not split symmetrically about unsplit line
- Resulting lines look qualitatively similar to normal Zeeman effect

~1 MG field in WD 0637+477

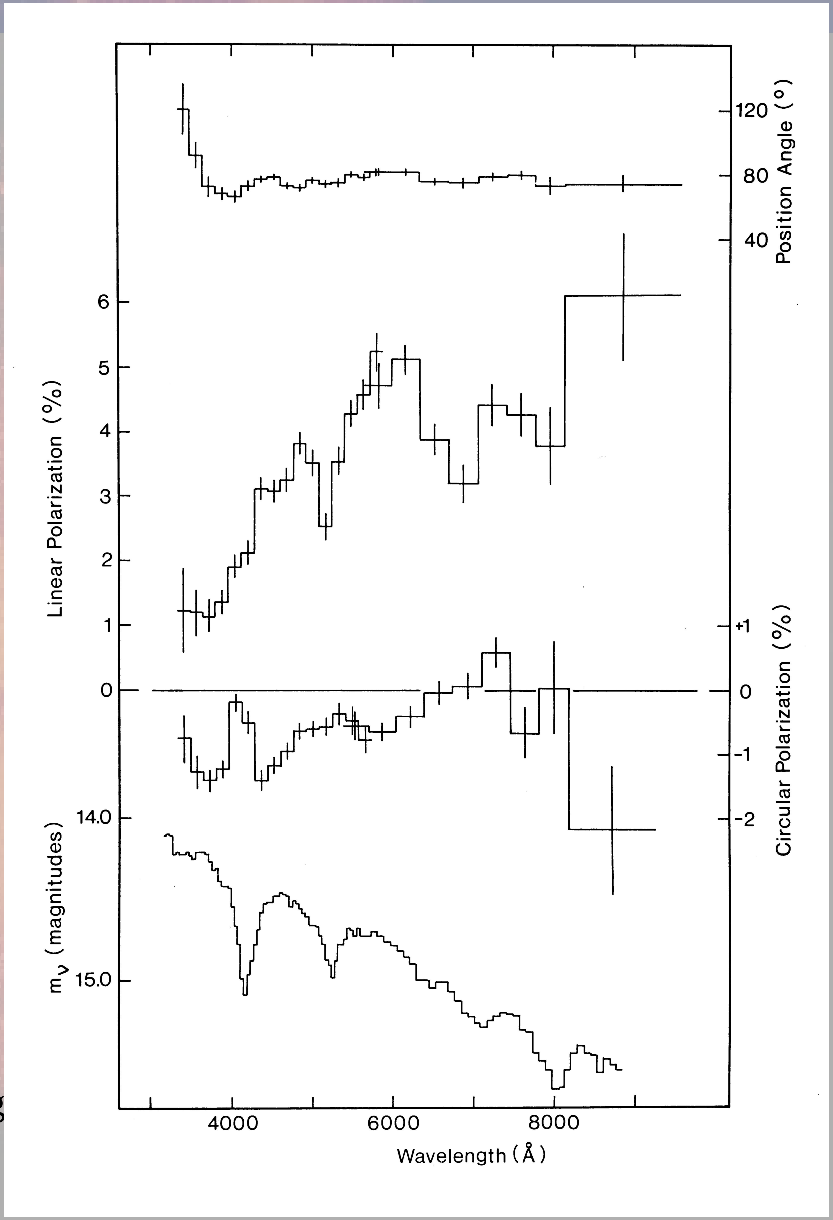


Continuum polarisation of white dwarf radiation in MG fields

- Physically, the fact that free electrons spiral around magnetic field lines in a particular sense means that the continuum absorption is *dichroic*. Right and left circularly polarised light will be absorbed *differently*, and the continuum radiation will be circularly polarised by a field that has a big component along the line of sight.
- Significant continuum circular polarisation is found above about 10 MG.
- For still larger fields (above about 100 MG) a similar effect produces linear polarisation of continuum radiation.
- However, it has not so far been possible to calculate polarisation spectra of continuum radiation that resemble observed polarisation spectra (cf. Koester & Chanmugam 1990, Rep. Prog. Phys., 53, 837, Sec 8).

White dwarf fields

- Magnetic fields have been found in over 50 isolated white dwarfs, and over 40 cataclysmic variable binaries
- These stars span a range in field strength from some kG to (probably) around 1000 MG
- They are either unvarying, or vary peridically with periods of hours or days.
- Weaker fields, below some 10s of MG, have more or less familiar spectral lines
- Above this field they have very strange I, V, and sometimes Q, U spectra – e.g. GD229 at right -->



Atomic structure in huge fields

- For fields above 10 MG, perturbation theory is no longer adequate to compute spectral line positions and splitting. The magnetic terms in the Hamiltonian are **comparable to** the Coulomb terms, and the structure of the combined system must be solved numerically.
- This is actually a very difficult problem. However, it has been done for H, and to a large extent for He. (For references consult e.g. Becken & Schmelcher 2002, Phys Rev A, 65, 033416)
- Basically, each line component decouples from the others and moves about in a dramatic way.
- Absorption lines in stellar spectra for fields over about 50 MG are affected by fact that the line positions vary rapidly with B, and B is not constant over the stellar surface. Lines occur at wavelengths where for some range of B the absorption wavelength does *not* change rapidly.

Precise calculations of hydrogen for large (~100 MG!) fields

- For large B values, the sigma components of spectral lines vary rapidly with wavelength. They are almost undetectable on stars where B varies by a factor of two.
- Some pi-like transitions have little variation over a range of field strength (“stationary components”). Such transitions can produce useful lines over a range of field strengths in the range of hundreds of MG (e.g. Wunner et al 1985, A&A 149, 102)

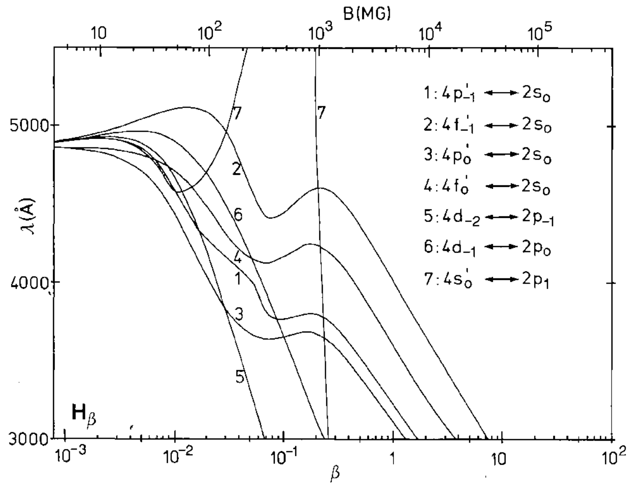


Fig. 3. The wavelengths of the 7 stationary $H\beta$ components as functions of the magnetic field

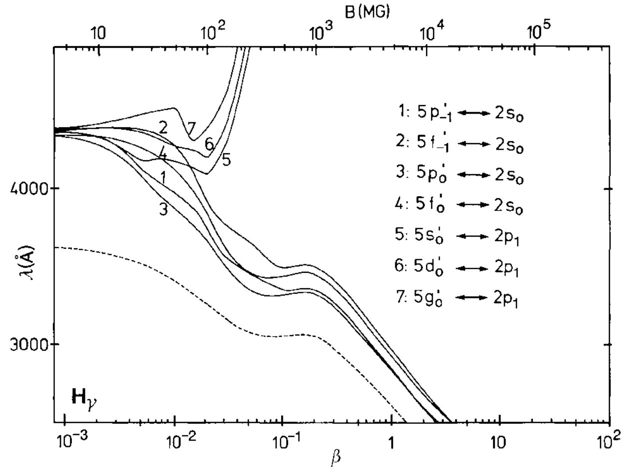


Fig. 4. The wavelengths of the 7 $H\gamma$ components stationary as functions of the magnetic field. Dashed curve: Balmer edge for transitions from $2s$ to the continuum

Modelling of white dwarf fields

- Three areas of modelling have been explored
- Low field stars can be modelled successfully using high-field Zeeman splitting theory (full magnetic Hamiltonian). The observed spectra are best fit with roughly dipolar fields, typically with some decentring (i.e. one pole stronger than the other) See e.g. Putney & Jordan 1995, ApJ 449, 863.
- In high field white dwarfs, the absorption features in the *I* spectrum can sometimes be fit with “stationary components” found in the theoretical spectral line wavelength computations. These models reveal fields of 100s of MG, and again are consistent with simple field structure.
- Efforts have been made to model the polarisation spectra These have been only somewhat successful for circular polarisation, and generally unsuccessful for linear polarisation (cf Putney 1999, ASP Conf 169, 195)
- Modelling suggests that fields are globally simple, approximately dipolar

Modelling of WD KUV 813-14

- Putney & Jordan 1995 have modelled this star with a field of about 45 MG, T_{eff} ~11000 K, and a variety of slightly different models
- I spectra at top, V spectra below; decentred dipole on left, dipole – quadrupole right

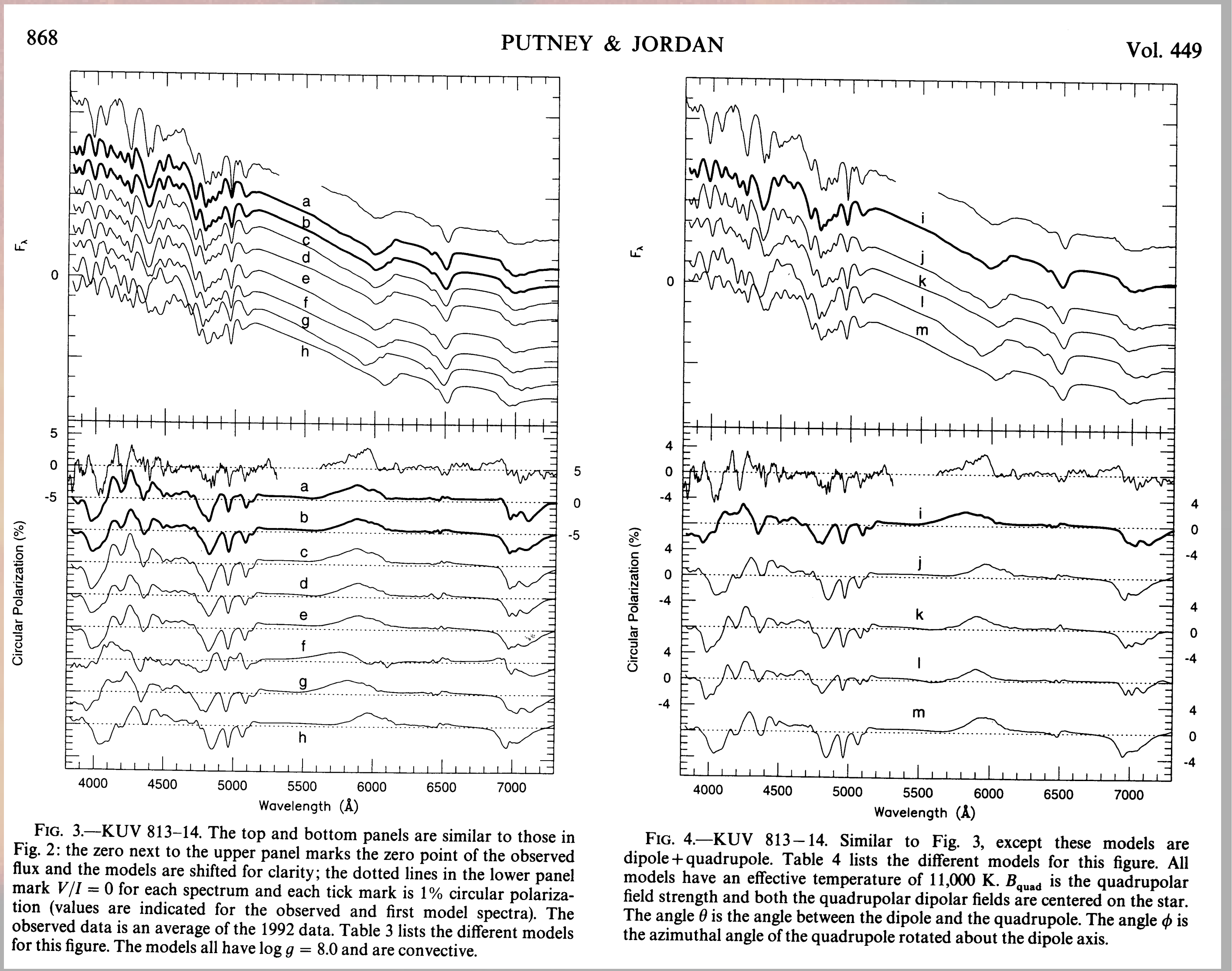


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FIG. 3.—KUV 813–14. The top and bottom panels are similar to those in Fig. 2: the zero next to the upper panel marks the zero point of the observed flux and the models are shifted for clarity; the dotted lines in the lower panel mark $V/I = 0$ for each spectrum and each tick mark is 1% circular polarization (values are indicated for the observed and first model spectra). The observed data is an average of the 1992 data. Table 3 lists the different models for this figure. The models all have $\log g = 8.0$ and are convective.

FIG. 4.—KUV 813–14. Similar to Fig. 3, except these models are dipole + quadrupole. Table 4 lists the different models for this figure. All models have an effective temperature of 11,000 K. B_{quad} is the quadrupolar field strength and both the quadrupolar dipolar fields are centered on the star. The angle θ is the angle between the dipole and the quadrupole. The angle ϕ is the azimuthal angle of the quadrupole rotated about the dipole axis.

Estimating field of Grw+70 8247

- Lower:
  I spectrum of WD with huge field
- Upper: varying positions of H absorption components with increasing *B*
- Absorption lines in *I* where components are stationary with *B*
- *B* ~ 300 MG

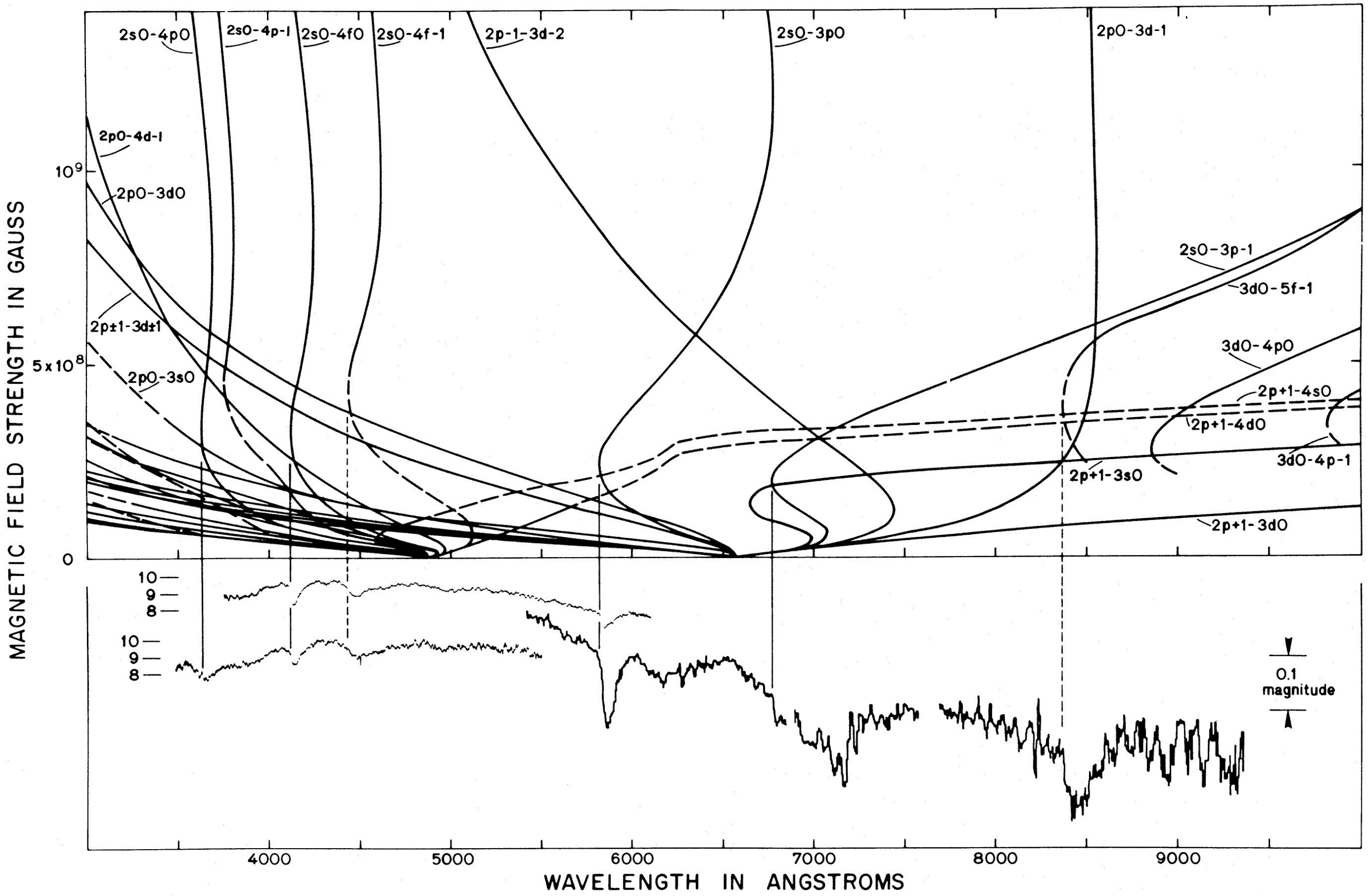

FIG. 1.—(*Below*) intensity spectra of Grw +70°8247, as described in text. (*Above*) Magnetically shifted component wavelengths from Rösner *et al.* (1984) as a function of magnetic field strength in gauss. Wavelength coincidences of "turnaround" components with absorption features in the spectra are indicated by vertical lines; dashed lines are used when component behavior has not yet been rigorously calculated. See § III*a*.

Fossil fields: tepid stars, white dwarfs

- Fossil fields are expected to have strengths which are not closely related to current rotation rates; in fact, because a long-lived field may help a star to lose angular momentum, strong fields might particularly occur in slowly rotating stars. This seems to be true of both magnetic Aps and white dwarfs.
- Such fields are also likely to change structure only on rather long time-scales. The lack of intrinsic variability (other than rotation of the host star) is an important argument that both Ap and white dwarf fields are probably fossil fields.
- Fossil fields, almost by definition, are produced during a previous phase of a star's life. This could be as recent as the pre-main sequence if a strong dynamo operated then, or could even be traced back to the field of the interstellar gas.