Synthesis of stellar spectra: a review of radiative transfer

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In the previous episodes....

- We explored the splitting of energy levels and spectral lines by the Zeeman (and Paschen-Back) effect, and found a number of useful tools
- We looked at methods of extracting useful estimates of longitudinal field $\langle B_z \rangle$ averaged over the visible stellar hemisphere from circularly polarised spectra of magnetic Ap stars, and we saw that in cases of large enough field and small enough v sin i we could also estimate the mean value over the hemisphere of $\langle B \rangle$
- Finally we looked at simple multipole field models that could reproduce the (limited) data from $\langle B_z \rangle$ and $\langle B \rangle$ observations over a stellar rotation
- Now we consider a much more powerful modelling method.

Why build models of spectra??

- Some information is available “directly” from spectrum (e.g. radial velocity, *v* sin *i*, presence of Fe or Eu, hemispheric average magnetic field strength).
- Simple models can be made to fit these simple data.
- Beyond simple results, spectrum synthesis is needed for
  - quantitative information, such as abundance of various chemical species, or distribution of magnetic field over surface
  - To test ideas about what is present on (or near) an unresolved star
- Basic idea: predict *spectrum* corresponding to hypothesis, and compare prediction with observed spectrum.
- This process is typically iterated to convergence – if possible!
- Discrepancies provide hints about missing physics in the model!

Basic process of (forward) spectrum modelling

- Two essential components are required, usually computed separately for practical reasons:
  - structure of outer layers of star from which radiation can escape (a “model atmosphere” - T(r), p(r), etc)
  - outgoing radiation field, usually described by “specific intensity” $I_\nu(\theta)$ of radiation (units: ergs/s-cm^2-Hz-sterad), to obtain details of spectrum of interest
- Structure is computed for most stars assuming that atmosphere is “thin” (plane-parallel), horizontally uniform, in hydrostatic equilibrium, and in a thermal steady state between hot stellar interior and cold exterior space
- Radiation emitted is computed from atmosphere model using “equation of radiative transfer”

Equation of radiative transfer: a simple introduction

- We start with the spectrum synthesis step, return to model atmospheres later
- Most complex and obscure element of this modelling process is equation of radiative transfer
- We start by looking at a very simple form of this equation, to build intuition, and then extend it to polarised radiation
- Basic idea is to look at specific intensity $I_\nu(z, \omega)$ of radiation passing through small volume of gas in a specific direction, and compute changes due to absorption and emission:

$$dI_\nu = \left(-\kappa_\nu I_\nu + j_\nu\right) ds$$

where κ_ν is the absorption coefficient (cm^2/cm^3) and j_ν is the emissivity per unit volume (ergs/s-cm^3-Hz-sterad)

EOT: simple introduction (2)

- Consider a stellar atmosphere with *ds* at an angle θ to the vertical, so $ds = dz/\cos\theta = dz/\mu$. Now define (monochromatic, vertical) optical depth $d\tau_\nu = -\kappa_\nu\, dz$. Then

$$\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - \frac{j_\nu}{\kappa_\nu} = I_\nu - S_\nu$$

  This is the equation of transfer (EOT).
- Some important points
  - This form ignores all scattering processes
  - In this approximation, the EOT describes the variation of radiation along a single ray. Each ray is independent of all others, and each has its own EOT
  - If we start a ray deep inside an object and follow it to the surface, this equation predicts the emerging light $I_\nu(0, \omega)$

EOT: simple introduction (3)

- In "local thermodynamic equilibrium" (LTE), S_ν is simply the Planck function $S_\nu = B_\nu(T) = (2h\nu^3/c^2)/[\exp(h\nu/kT) - 1]$
- If we multiply the equation of transfer by the integrating factor $\exp(-\tau_\nu/\mu)$, and assume that S_ν is a *known* function of τ_ν, the EOT can be integrated:

$$I_\nu(\tau_\nu) = e^{\tau_\nu/\mu} \int_{\tau_\nu}^{\infty} S_\nu(\tau_\nu') e^{-\tau_\nu'/\mu} d\tau_\nu'/\mu$$

- With lower limit 0, we get specific intensity emerging from surface.
- Now suppose that $S_\nu(\tau_c) = S_0(1 + \beta\tau_c)$, for τ_c of a particular (continuum) frequency and nearby frequencies. Integrating:

$$I_c(0) = S_0(1 + \beta\mu) = S_c(\tau_c = \mu)$$

EOT: simple introduction (4)

- Now suppose that in a spectral line $\kappa_\nu = \kappa_c + \kappa_{line} = (1+\eta_\nu)\kappa_c$ where η_ν is a function of frequency but not depth.
- Then $\tau_\nu = (1+\eta_\nu)\tau_c$ and $S_\nu(\tau_\nu) = S_0[1+\beta\tau_\nu/(1+\eta_\nu)]$ so finally we get an expression for surface intensity
  $$I_\nu(0) = S_0[1+\beta\mu/(1+\eta_\nu)]$$
- This shows that if we know how to write the line absorption coefficient as a function of ν or λ (normally as a Voigt profile), then we see that as we get near line centre the emergent intensity drops, while in the nearby continuum it has the continuum value found on previous slide.
- The point: in this case the EOTs (one for each λ) are *simply* solvable first order, linear DEs with driving terms. (Of course, there are a lot of more complex situations!)

EOT: simple introduction (5)

- The expression we have just found for the specific intensity $I_\nu(0,\omega) = S_0 [1 + \beta\mu/(1 + \eta_\nu)]$ contains everything(!) : effects of line opacity, slope of source function, and limb darkening
- It is important to realise that $I_\nu(0,\omega)$ is the local emission from a small area at the top of the atmosphere of a star, at a particular angle with respect to the local vertical.
- To find the stellar flux we would measure here on Earth from the star, we have to add together all the contributions from various points on the visible hemisphere, each computed for the direction towards Earth
- Rays from the centre of the stellar disk aimed at Earth emerge nearly vertically; rays from near the limb come out at larger angles
- To get a spectrum, we repeat this addition for many adjacent wavelengths or frequencies

The Stokes vector

- Now recall that to described *polarised* light mathematically we use the Stokes parameter description: *[I, Q, U, V]*
- *I* is the total intensity of light in the beam
- For *Q* and *U*, measure the intensity of the beam through perfect linear polarisers (polarising analysers) orientated at 0, 45, 90, and 135 degrees. $Q = I_0 - I_{90}$, $U = I_{45} - I_{135}$.
- Measure the intensity of the beam thorough two perfect circular polarisers. $V = I_{right} - I_{left}$.
- These four quantities adequately describe the polarisation state of a light beam
- *[I, Q, U, V]* are functions of frequency and direction
- *Q, U, V* are sometimes normalised to *I (Q -> Q/I*, etc).

Equation of radiative transfer polarised radiation

$$\mu \frac{dI}{d\tau_\nu} = (1+\eta_I)(I-B_\nu) + \eta_Q Q + \eta_V V$$

$$\mu \frac{dQ}{d\tau_\nu} = \eta_Q (I-B_\nu) + (1+\eta_I) Q - \rho_R U$$

$$\mu \frac{dU}{d\tau_\nu} = \rho_R Q + (1+\eta_I) U - \rho_W V$$

$$\mu \frac{dV}{d\tau_\nu} = \eta_V (I-B_\nu) + \rho_W U + (1+\eta_I) V$$

- For polarised radiative transfer, EOTs are written in terms of Stokes parameters
- First order linear DEs like 1D EOT, but now four coupled equations, one for each Stokes parameter
- No scattering is included
- In LTE, B_ν is Planck function
- Factors η describe absorption, factors ρ describe retardation (anomalous dispersion)

Coupling coefficients in EOTs

- Define η_p, η_l, η_r as the ratios of line (Voigt) profile opacity, in pi, left sigma, and right sigma Zeeman components, to the continuum opacity. Then

$$\eta_I = 0.5\,\eta_p \sin^2\psi + 0.25(\eta_l + \eta_r)(1 + \cos^2\psi)$$
$$\eta_Q = 0.5\,\eta_p - 0.25(\eta_l + \eta_r)\sin^2\psi$$
$$\eta_V = (\eta_r - \eta_l)\cos\psi$$

  Projection of field defines vertical (*Q*), and ψ is the angle between field and vertical.
- Similar expressions involving Faraday-Voigt function are required for the retardance terms
- Note that η_Q and η_V are differences of Zeeman opacity coefficients, like *Q* and *U*. They act to introduce polarisation.

Comments on polarised EOTs

- For given atmospheric structure, we can start the 4 polarised EOTs with unpolarised black-body radiation at great depth, follow many rays (many directions, frequencies) to surface and compute (discrete approximation of) emergent flux
- Equations describe effects of polarised absorption and retardation on outflowing radiation
- For non-degenerate stars, polarisation is introduced essentially by polarised Zeeman line components
- Cannot accurately predict intensity or polarisation of emergent stellar spectrum without solving these equations – even intensity spectrum is substantially different from what is computed with a single unpolarised equation of transfer

Analytical solutions

- As for the unpolarised equation of transport, there is an analytic solution to the simplest case of the polarised equations assuming a linear source function and a normal Zeeman triplet (worth playing with to build intuition)
- Simplest form was derived by Unno (1956, PASJ 8, 108) in paper which laid the basis for setting up EOT in terms of Stokes parameters. Note that this paper ignores retardation.

$$\frac{I_0(0,\theta)-I(0,\theta)}{I_0(0,\theta)}=\frac{\beta\cos\theta}{1+\beta\cos\theta}\left[1-\frac{1+\eta_I}{(1+\eta_I)^2-\eta_Q^2-\eta_V^2}\right]$$

$$\frac{V(0,\theta)}{I_0(0,\theta)}=\frac{-\beta\cos\theta}{1+\beta\cos\theta}\left[\frac{\eta_V}{(1+\eta_I)^2-\eta_Q^2-\eta_V^2}\right]$$

- See also Martin & Wickramasinghe 1979, MNRAS 189, 883

Building a model atmosphere

- Same EOT (or EOTs) are used for building model atmosphere. Usual requirements imposed
  - Hydrostatic equilibrium, one-dimensional (flat)
  - Scattering is ignored, or treated as absorption
  - LTE (source function is Planck function everywhere)
  - Integrated flux is constant through atmosphere (although detailed frequency distribution changes from level to level; this determines temperature structure
  - (At small τ_c where flux not affected much by absorption, local conditions are computed using energy balance)
  - Results depend strongly on good treatment of continuum and line opacity, and on abundance table assumed

Available atmosphere codes & models

- Model atmospheres may be computed using either unpolarised or polarised transfer.
- Until recently, only unpolarised model atmospheres were available
  - Hot stars: ATLAS (cf http://kurucz.harvard.edu/ , especially grids and programs)
  - Cool stars: MARCS models (B Gustafsson et al; see also http://www.uwosh.edu/mike/exercises/marcs/marcs.html)
  - Wide range: Phoenix models (P Hauschildt et al)
- D Shulyak and S Khan have recently developed LLModels, a code for hot stars that can compute models including magnetic polarisation effects and arbitrary abundance tables
- It mostly seems to be an acceptable approximation to use unpolarised model atmospheres, or ones with simple line splitting, but the appropriate abundance table has important effects.