

Atomic physics needed to measure fields in stars



JDL

Atom in a magnetic field

- Basic idea is that a magnet in a magnetic field \mathbf{B} has an *orientation energy* $W = \boldsymbol{\mu} \cdot \mathbf{B}$, a (magnetic moment) $\times B$
- Easy to get estimate of effect: in an atom, $mv^2/r \sim e^2/r^2$ and $mvr \sim \hbar$, which leads to $r \sim \hbar^2/me^2 = a_0$ and $v \sim e^2/\hbar$.
- Then magnetic energy is $W \sim (evB/c)a_0 \sim (e\hbar/mc)B$, and magnetic moment of an electron must be of order $(e\hbar/mc)$
- $(e\hbar/2mc)$ is called the Bohr magneton. It is magnetic moment of electron spin or of a single unit of electron orbital angular momentum.
- This W is characteristic amplitude of perturbation of an atomic energy level by interaction between field and the effective electric current produced by orbiting or spinning electron

Atomic Hamiltonian with B

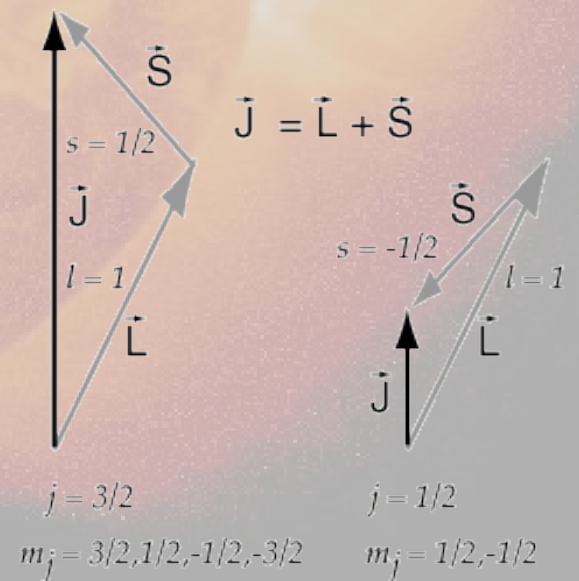
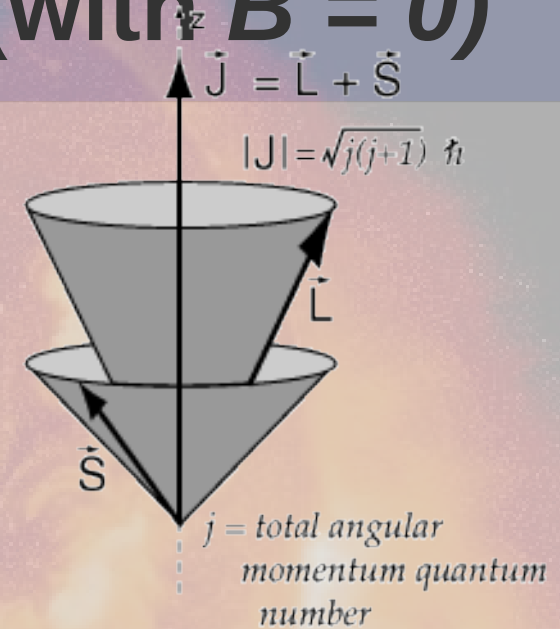
- Now consider the Hamiltonian of an atom in a magnetic field (Gaussian cgs units; atom in L-S coupling)

$$H = -\frac{\hbar^2}{2m} \nabla^2 + V(r) + \xi(r) \mathbf{L} \cdot \mathbf{S} + \left[-\frac{e}{2mc} \mathbf{B} \cdot (\mathbf{L} + 2\mathbf{S}) + \frac{e^2}{8mc^2} B^2 r^2 \sin^2 \theta \right]$$

- First 3 terms are kinetic energy, electronic potential, spin-orbit coupling with $\xi(r) = (1/2 m^2 c^2) (1/r) (dV/dr)$
- Last two terms are magnetic energy terms derived from magnetic vector potential
- For fields up to $B \sim 10$ MG (1 kT), magnetic terms are small compared to Coulomb potential. Fine structure and effects of field are treated by perturbation theory

Recall atomic structure due to spin-orbit coupling in atoms (with $B = 0$)

- In a light atom, all unpaired orbital angular momenta (of electrons in unfilled shells) tend to couple together, and all unpaired spins.
- Different orientations of \mathbf{L} relative to \mathbf{S} lead to different (but nearby) energy levels
- We label such states by $^{2S+1}L_j$, for example $^2P_{3/2}$ or $^2P_{1/2}$, as in the upper levels of the Na D lines.



Magnetic field regimes

- Now return to that Hamiltonian again

$$H = -\frac{\hbar}{2m} \nabla^2 + V(r) + \xi(r) \mathbf{L} \cdot \mathbf{S} + \left[-\frac{e}{2mc} \mathbf{B} \cdot (\mathbf{L} + 2\mathbf{S}) + \frac{e^2}{8mc^2} B^2 r^2 \sin^2 \theta \right]$$

- (Perturbation theory) regimes:
 - Quadratic magnetic term \ll linear term \ll spin-orbit term: (linear) Zeeman effect
 - Quadratic magnetic term \ll linear term and spin-orbit term \ll linear term: Paschen-Back effect
 - Quadratic magnetic term \gg linear term and quadratic term \gg spin-orbit term: quadratic Zeeman effect
- See Schiff 1955, Quantum Mechanics, Secs 23 & 39

Field range of various regimes

- May estimate size of magnetic terms by taking $L \sim \hbar$, $r \sim$ Bohr radius a_0 , $V \sim Ze/r$. We find
 - For normal atoms and $B < 50$ kG (5 T), most atomic lines are in Zeeman regime. This is usual situation for non-degenerate stars.
 - Fine structure splitting varies greatly from one level to another, so a few lines may be in Paschen-Back limit at much smaller B than normal (e.g. Fe II 6147 and 6149 Å). Paschen-Back splitting occurs in H and Li below 30 kG
 - Above about 100 kG quadratic term becomes important. Quadratic Zeeman effect is observed in lines of H in WDs
 - Above about 10 MG magnetic terms become comparable to Coulomb term, perturbation methods no longer work. Must solve structure of atom in combined field

Zeeman effect

- In weak-field (Zeeman) limit, atomic energy level is only slightly perturbed by $(e/2mc) \mathbf{B} \cdot (\mathbf{L} + 2\mathbf{S})$
- In L-S coupling (atoms up to Fe peak), J and m_J are good quantum numbers. Magnetic moment of atom is aligned with \mathbf{J} . Energy shift of level proportional to $\mathbf{B} \cdot \mathbf{J}$, so there are $2J+1$ different magnetic sublevels

$$E_i = E_{i0} + g_i (e/2mc) B m_J \hbar$$

where g_i is (dimensionless) Lande factor (in L-S):

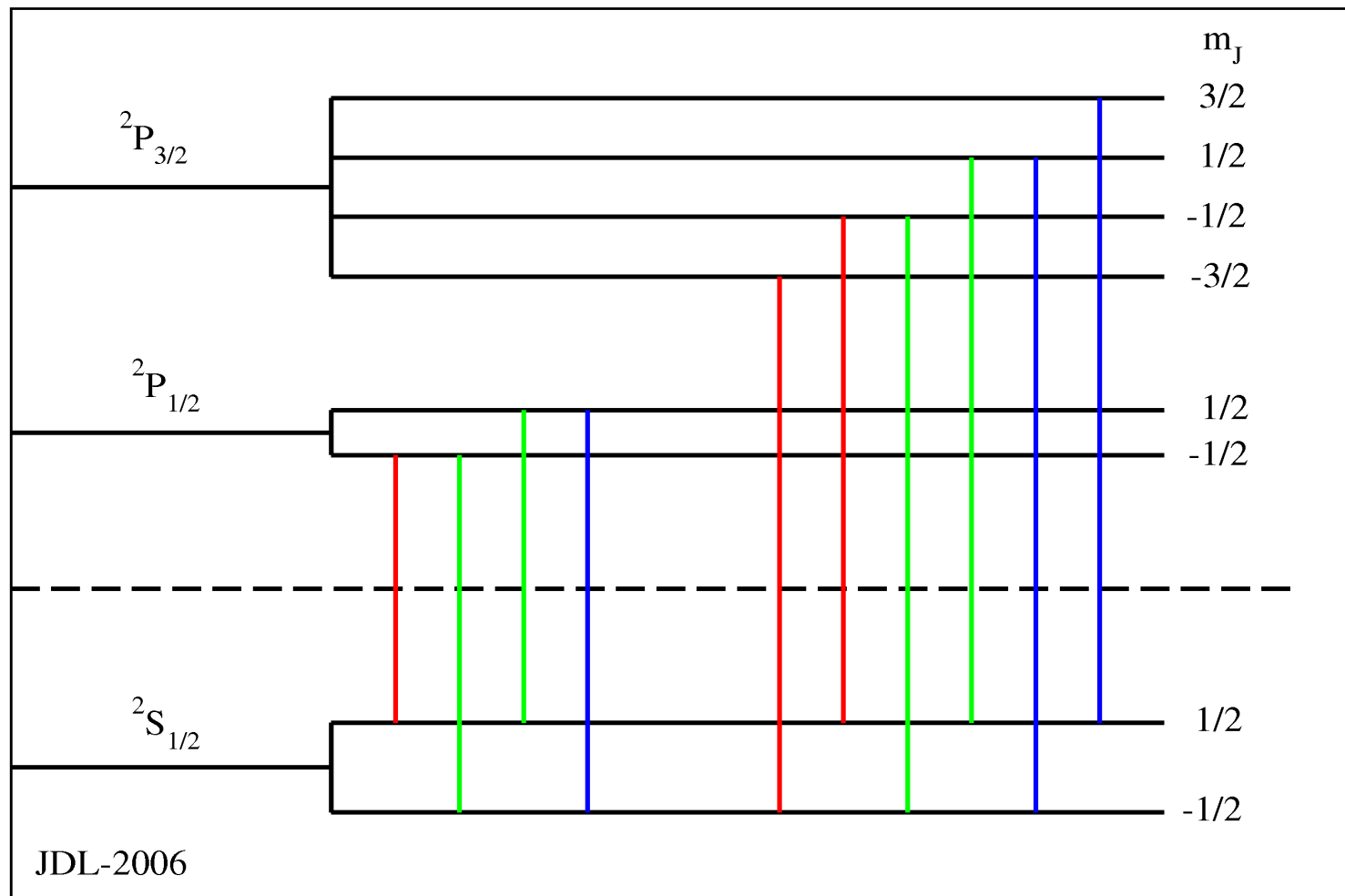
$$g_i = 1 + \left[J(J+1) + S(S+1) - L(L+1) \right] / \left[2J(J+1) \right]$$

- Spectral lines arise as (allowed) differences between sublevels of lower and upper energy levels: $\Delta m_J = 0$ (π), -1 or $+1$ (σ)

Example: Zeeman line components

Zeeman components for sodium D lines

green: pi components, red & blue: sigma components



Subcomponent separation due to Zeeman effect

- $$E_i = E_{0i} + g_i \left(\frac{e}{2mc} \right) B m_i \hbar$$
$$\nu_{ij} = \frac{(E_i - E_j)}{h} = \nu_0 + \left(\frac{e \hbar B}{2mc h} \right) (g_i m_i - g_j m_j)$$
$$\lambda_{ij} = c / \nu_{ij} = \lambda_0 + \Delta \lambda_Z = \lambda_0 + \left(\frac{e \lambda_0^2 B}{4\pi m c^2} \right) (g_j m_j - g_i m_i)$$
$$\Delta \lambda_Z (\text{\AA}) = 4.67 \cdot 10^{-13} \lambda_0^2 B (g_j m_j - g_i m_i)$$
- Note that splitting is (1) proportional to B and to λ^2 , and (2) is about +/- 0.012 A (π - σ separation) for 5000 A and 1 kG

Zeeman patterns

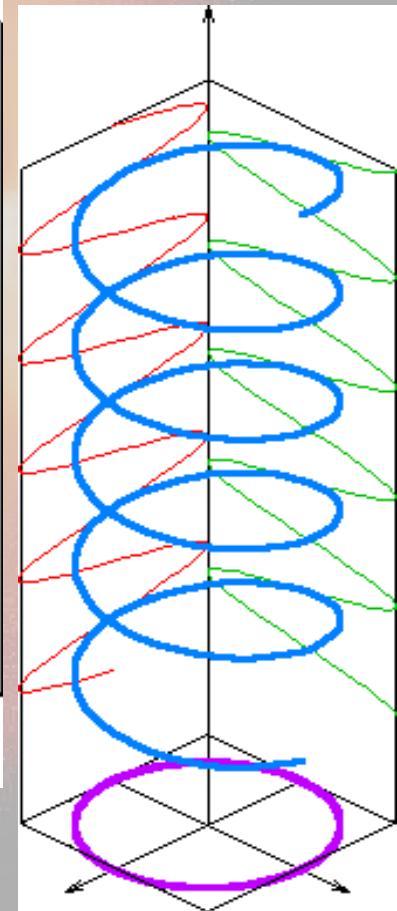
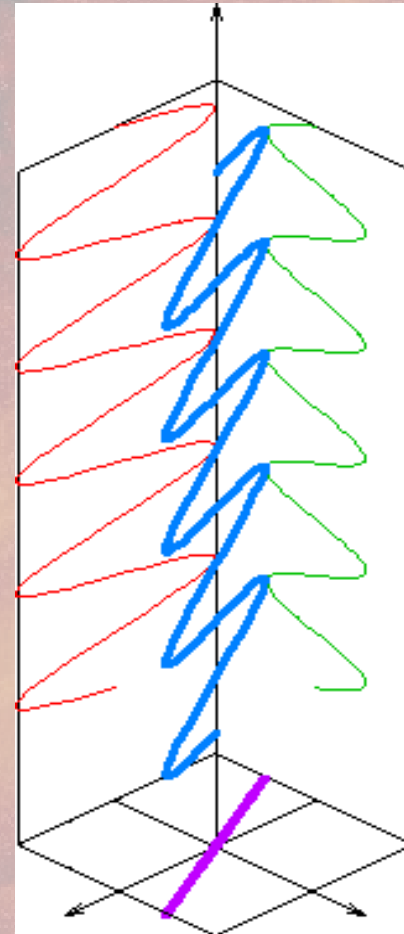
- Allowed transitions have $\Delta m_j = 0$ (π), -1 or $+1$ (σ). Only some combinations of sublevels produce lines!
- If spacing of upper sublevels is same as spacing of lower sublevels, only three line components appear (“normal Zeeman effect”)
- Usually spacing of upper sublevels is different from spacing of lower sublevels, and several lines of each of $\Delta m_j = 0, -1$ or $+1$ are present (“anomalous Zeeman effect”)
- A few transitions have no splitting (“null lines”)
- Splitting of each level determined by g_i values. Best values are from experiments (see Moore, Atomic Energy Levels, NBS Publication 467), or from atomic energy level calculations, but L-S coupling values are often good

Paschen-Back Effect

- This regime has few astronomical applications – most fields in non-degenerate stars are too weak to push lines into Paschen-Back range
- A few pairs of levels have very small fine-structure separation, and their Zeeman patterns are distorted by “partial” Paschen-Back effect, e.g. Fe II 6147-49 Å
- In Paschen-Back regime, **L** and **S** decouple, so J is not a good quantum number. Now m_L and m_S become good quantum numbers, so perturbation energy becomes
$$\left(\frac{e}{2mc}\right) B (m_L + 2 m_S) \hbar$$
- With this perturbation, all lines are split by same amount. Only three components ($\Delta m_L = -1, 0, 1$) are seen

Polarisation and properties of Zeeman components

- Another property of Zeeman line components that turns out to be very important in practice is their *polarisation*
- “Natural” light may be thought of classically as made up of short wave trains that have no particular phase or plane of polarisation
- Linearly polarised light has the electric field vector confined to a single plane (left figure)
- Circularly polarised light has an electric field vector which at any instant is spiral in form (right)



Polarisation of Zeeman components

- Polarisation of Zeeman line components depends on orientation of field to line of sight
- For field transverse to line of sight, $\Delta m_j = 0$ (π) components in an *emission* line source are polarised linearly, parallel to field lines, while $\Delta m_j = -1$ or $+1$ (σ) components are perpendicular to field
- For field parallel to the line of sight, $\Delta m_j = 0$ components *vanish*, while the $\Delta m_j = -1$ or $+1$ components have opposite circular polarisations
- Intermediate field orientations: elliptical polarisation

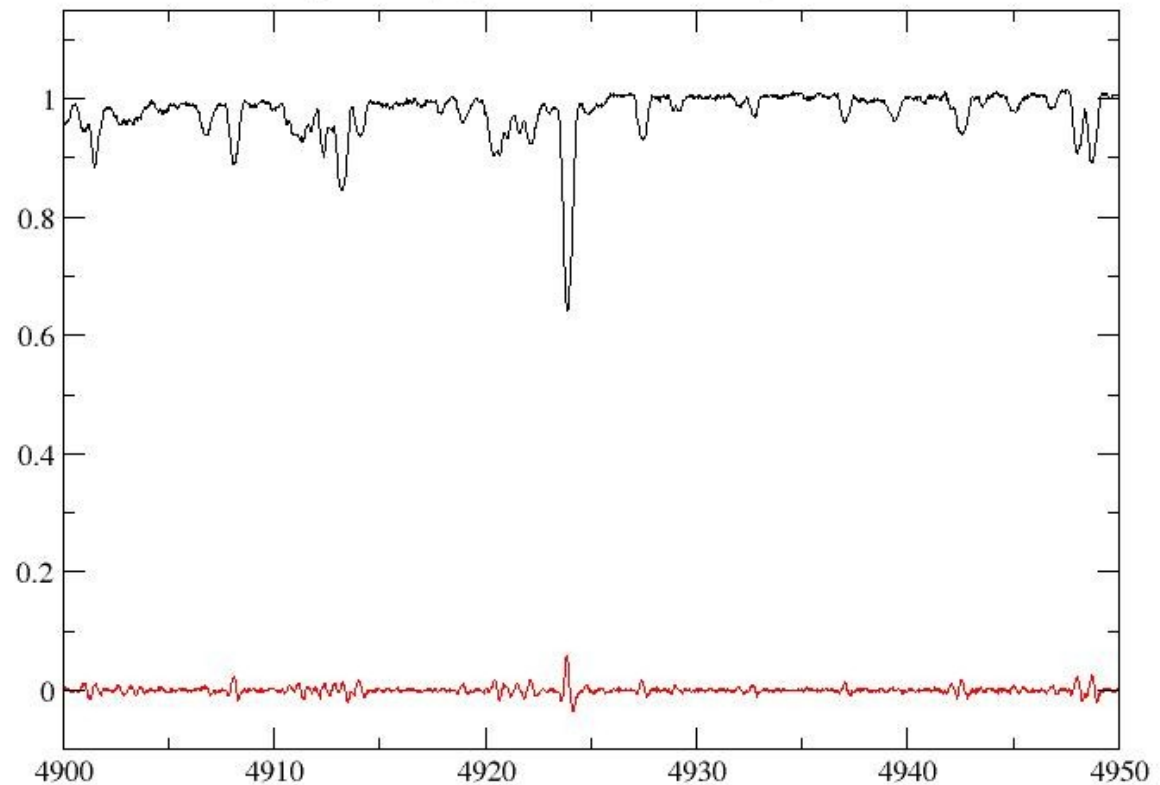
Detection of fields in stars

- Zeeman splitting is a clear effect, and allows measurement of total field. Why should we be interested in obscure polarisation properties??
- Zeeman splitting in stars (typically 0.01 Å per kG) can be masked by even small $v \sin i$ of a few km/s
- In fact, Zeeman splitting is clearly observed in only a small number of upper main sequence stars, and essentially never in low mass main sequence stars or giants
- Polarisation turns out to provide a much more powerful, and widely useful, means of detecting fields

Line splitting (not) and polarisation in strongly magnetic star HD 112413

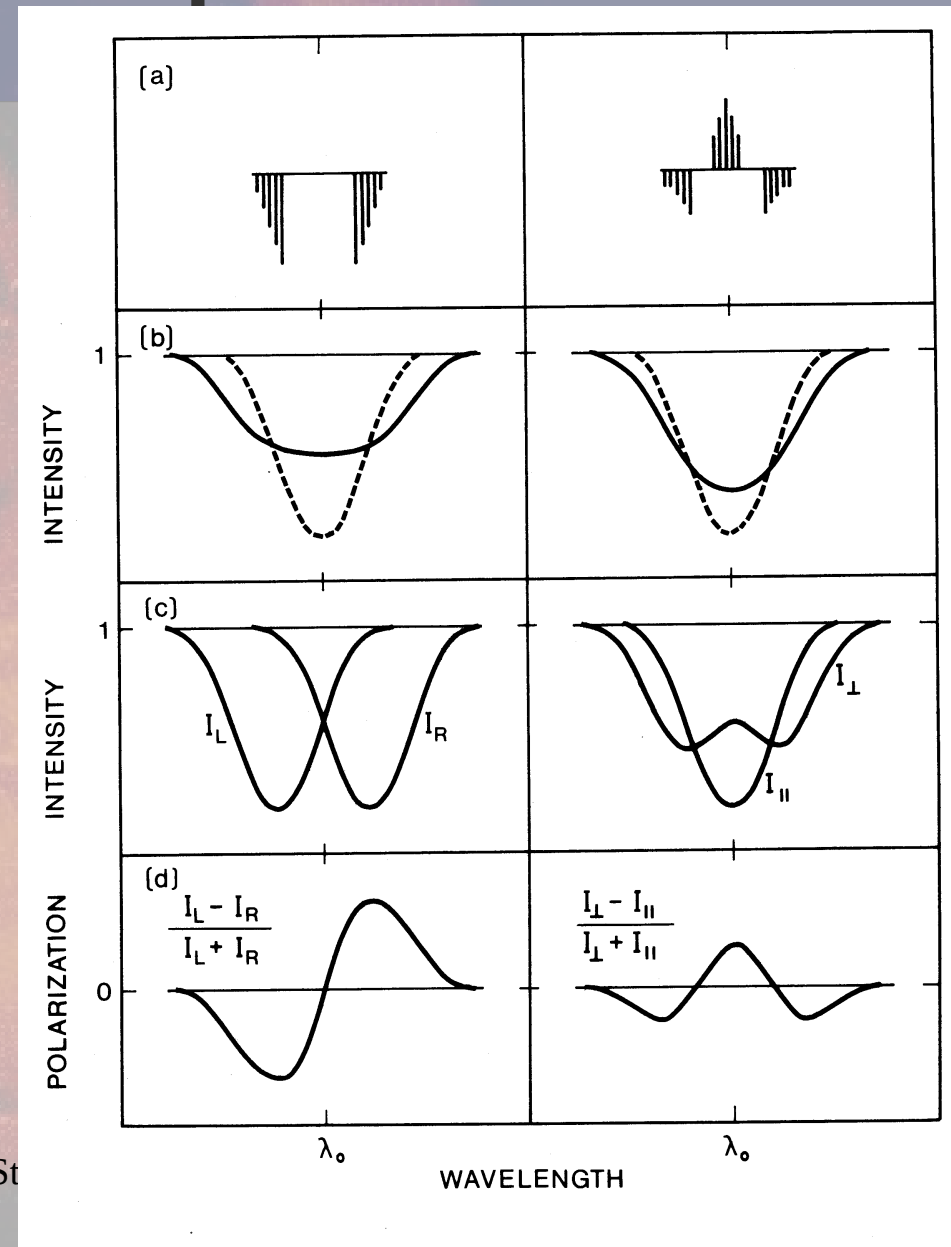
HD 112413 = alpha 2 CVn

Top: intensity spectrum; bottom: circular polarisation



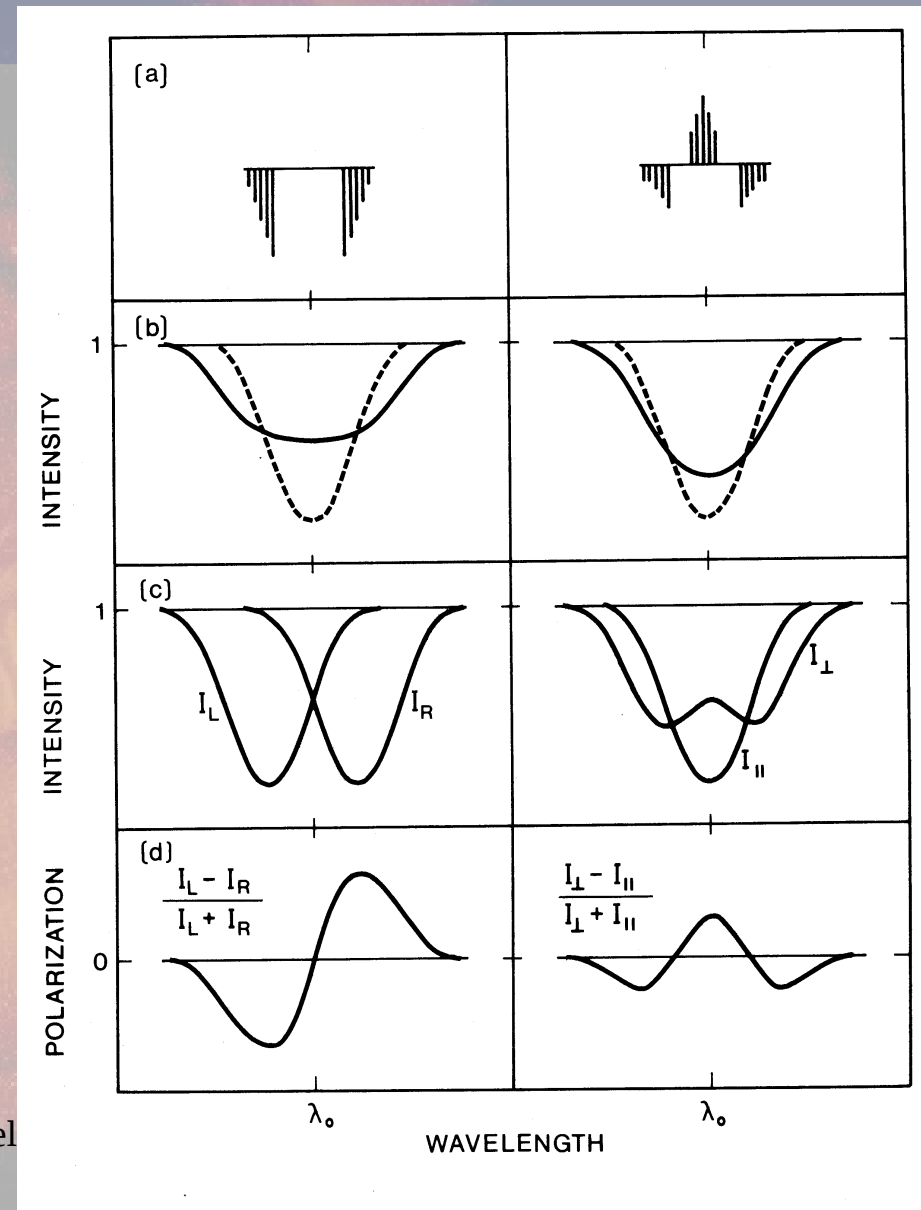
Polarisation in stellar spectral lines

- In figure, left column is for field along line of sight, right is for field transverse to sight line
- Intermediate field orientations give intermediate results
- Top row shows splitting of a particular line in laboratory
- Next row shows effects on unpolarised absorption line – little change in shape, and in any case not a distinctive magnetic signature



Polarisation in stellar spectral lines

- Third row: absorption in left and right circular polarisation (left), two orthogonal linear polarisation directions (right)
- Bottom row is difference spectra (net polarisation vs wavelength) – note large (1st order) effect in circular polarisation



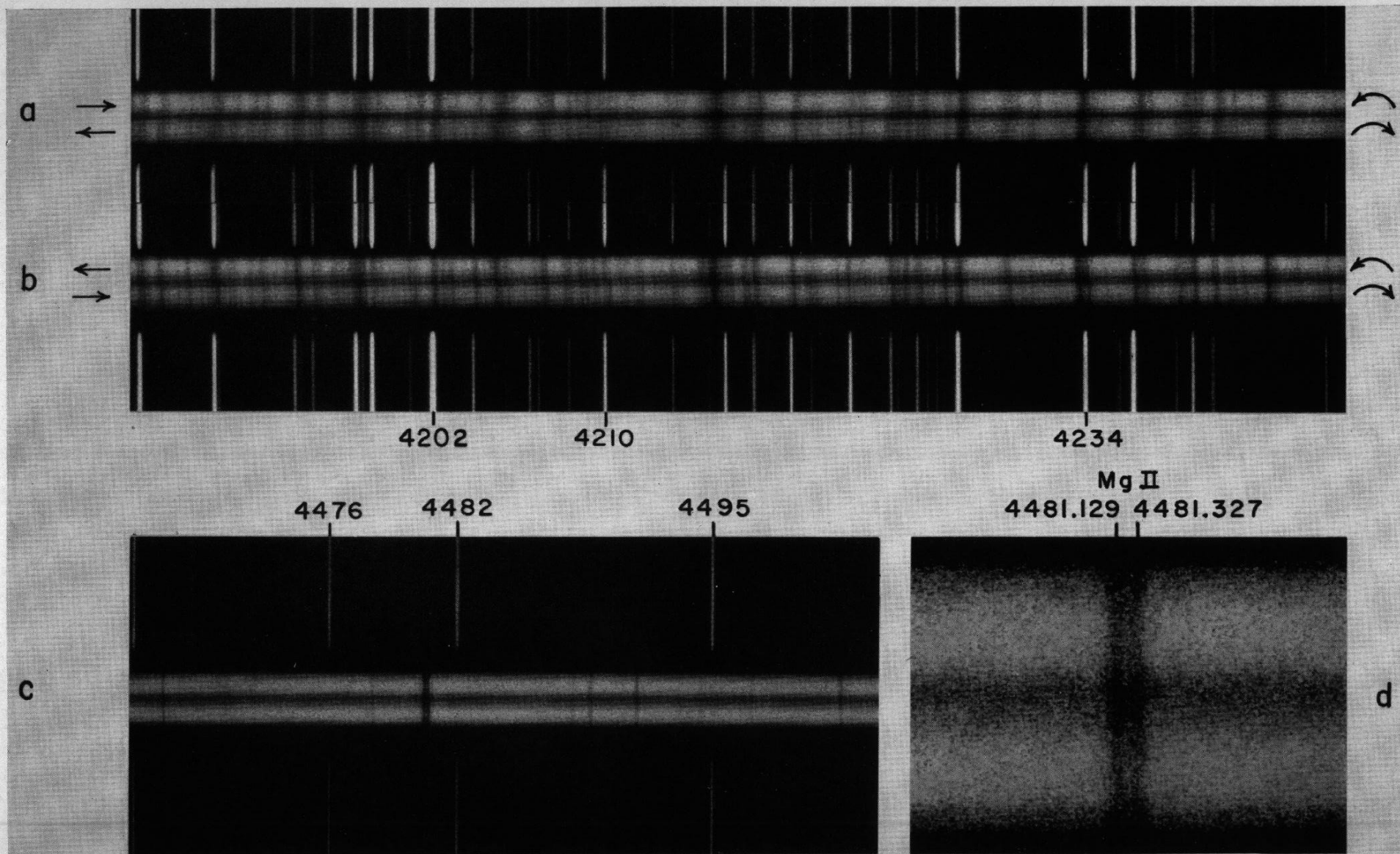


FIG. 2.—*a* and *b* are double spectra of the magnetic variable 53 Cam (No. 28), taken near the phases of maximum negative and positive field intensity, respectively. The two stellar spectra on each plate are taken through the differential analyzer for right- and left-handed polarization. Note the differential offsets of the lines, which are proportional to the field intensity, and which are of opposite sign for *a* and *b*. The lines (note particularly Eu II λ 4205) are much sharper in *b*. The double spectrogram *c* of the ultra-sharp-line, weak-field, "manganese star" ι CrB (No. 57) illustrates the sharpness of the lines ($w=0.07$). The resolution of Mg II λ 4481 into its two components about 0.2 Å apart in the spectrum of ι CrB is well shown in *d*, which is enlarged about sixty times from the original plate.

Field measurements in Ap stars: simple approximations

- Objective is to deduce some value of magnetic field strength from spectrophotometry, such as $\langle B_z \rangle$ “mean longitudinal field”, or $\langle B \rangle$ “mean field modulus” to describe result of detection of an atomic signal of magnetic field
- Two simple approximations have been used for many measurements:
 - Weak line limit
 - Weak field limit
- Weak line limit is approximately correct for most spectral lines, although it leads to somewhat different results from strong and weak lines
- Weak field limit is good up to about 1 kG for most metal lines, or up to at least 10 kG for Balmer lines

Weak line approximation

- Objective is to use circular polarisation observations to derive a mean line-of-sight field strength for observed star (or observation of line splitting for mean field modulus)
- In an optically thin gas, the separation between the position of the spectral line as seen in right and in left circularly polarised light is given by the mean separation of the sigma components multiplied by the cosine of the angle θ between the field and the line of sight

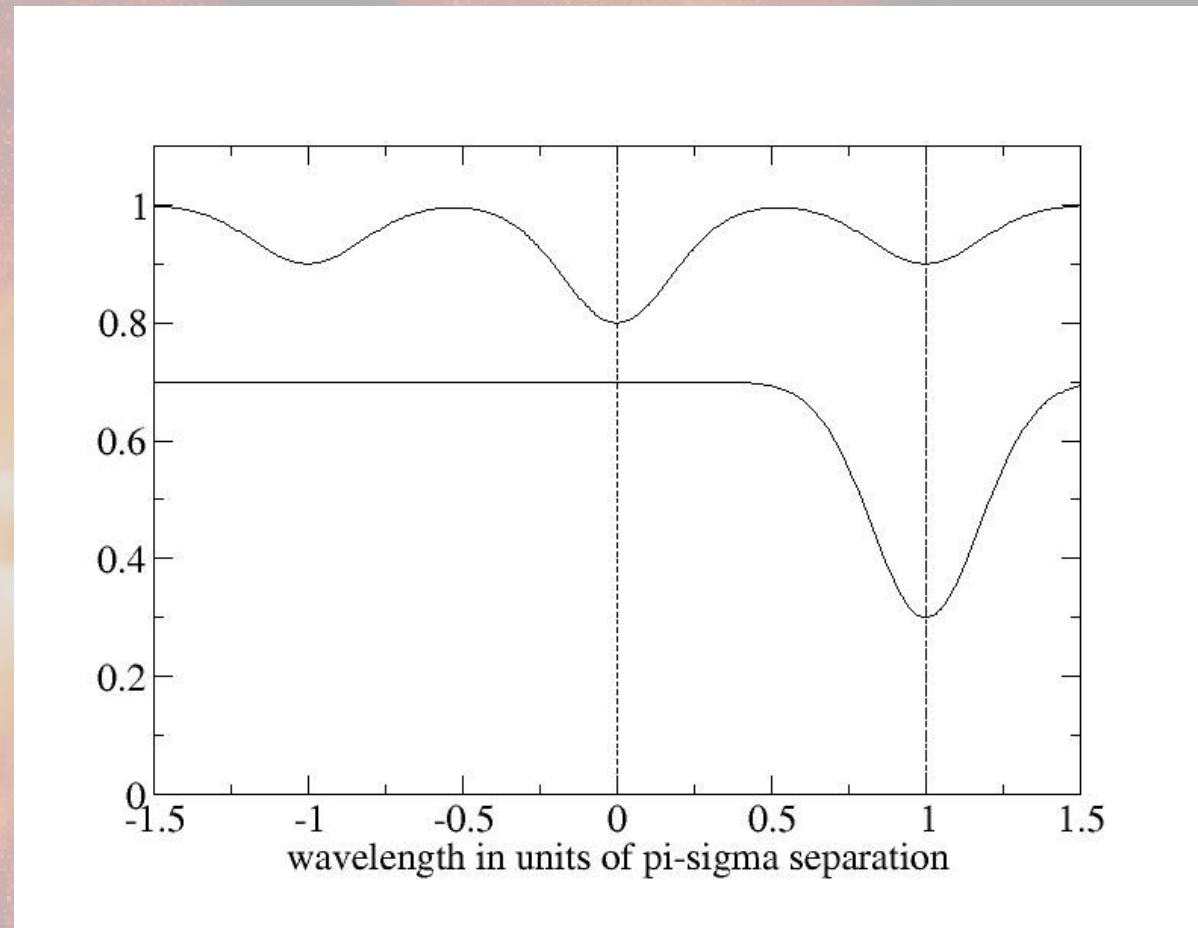
$$2 \Delta \lambda_z (\text{\AA}) = 2 \times 4.67 \times 10^{-13} \lambda_0^2 z \langle B \cos \theta \rangle$$

where z is a suitable average Lande factor

- The weak line approximation consists of assuming that this is correct for spectral lines even if they are strong enough to be saturated

Zeeman pattern in one sense of circularly polarised light

- Figure shows Zeeman pattern seen in *one* sense of circularly polarised light, in transverse field (top) and longitudinal field
- As direction of field shifts from transverse to longitudinal, mean position of observed line moves away from non-magnetic position

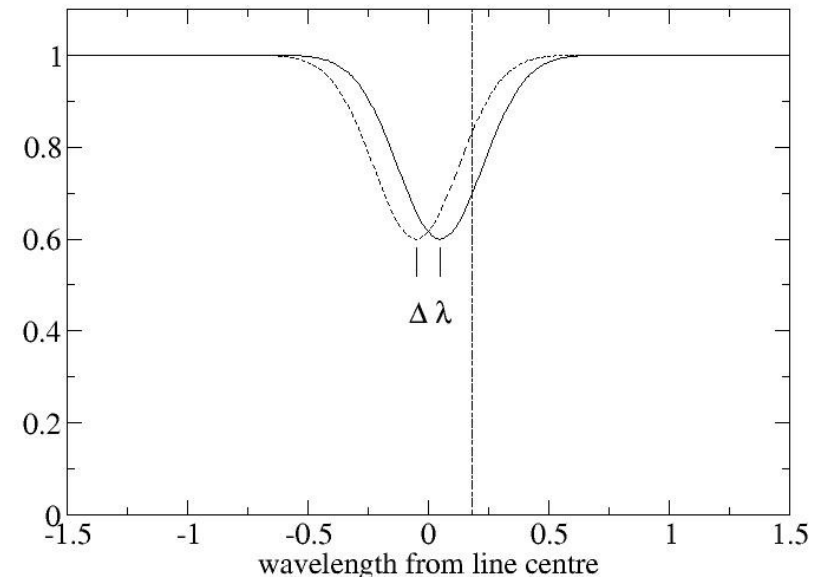


Field measurement in Ap stars weak field limit

- If the field is very weak, so splitting is small compared to (local) line width, the separation of the two circularly analysed components is as shown below. A measurement of net circular polarisation V in the line wing yields a field estimate through

$$V(\lambda) \approx 4.67 \cdot 10^{-13} z \lambda^2 (dI/d\lambda) \langle B_z \rangle$$

where all wavelengths are measured in Å.
(cf Landstreet 1982, ApJ 258, 639; Bagnulo et al 2002, A&A 389, 191)



Measuring the mean field modulus

- If star has small rotation velocity (less than few km/s) and a large field (some kG), the Zeeman splitting of (at least some) spectral lines may be measurable
- Example of effect in HD 94660 (Mathys)
- Splitting is proportional to total magnitude of field, so we obtain a surface-averaged value of $\langle B \rangle$
- These data provide quite different constraints on nature of stellar magnetic field than $\langle B_z \rangle$, and help make possible more realistic modelling of observed fields.

Zeeman splitting in HD 94660

